6. WGS 84 GEOID

6.1 General

In geodetic applications, three different surfaces or earth figures are normally involved. In addition to the earth's natural or physical surface, these include a geometric or mathematical figure of the earth taken to be an equipotential ellipsoid of revolution (Chapter 3), and a second equipotential surface or figure of the earth, the geoid. The geoid is defined as that particular equipotential surface of the earth that coincides with mean sea level over the oceans and extends hypothetically beneath all land surfaces. In a mathematical sense, the geoid is also defined as so many meters above (+N) or below (-N) the ellipsoid (Figure 6.1). In Figure 6.1 and in subsequent figures and equations, the difference in direction between the geodetic normal and the vertical is ignored due to its insignificance to the topic discussed here.

The first definition of the geoid has great practical importance since it refers to the hypothetical extension of mean sea level (the geoid) beneath land surfaces. In this capacity, the geoid serves not only as the vertical datum (reference surface) for height-above-mean sea level (h) values in areas where elevation data is not available from conventional leveling, but is fundamental to the determination of the h-values themselves. This is apparent from the Equations

$$H = N + h \tag{6-1}$$

$$h = H - N \tag{6-2}$$

and Figures 6.2 and 6.3, where:

H = geodetic height = height above the ellipsoid*

N = geoid height

h = height-above-mean sea level.

Figures 6.2 and 6.3 illustrate the use of geoid heights in the determination of h-values from geodetic heights derived using satellite receivers (e.g., NAVSTAR GPS receivers) located on the earth's physical surface and aboard a vehicle operating near the earth (or in space), respectively.

In particular, it is easy to see from Figure 6.2 how geoid heights (N) position the geoid with respect to the ellipsoid where the former can then serve as the vertical datum for the height-above-mean sea level values which the N-values also helped determine. It is hard to over-estimate the importance of the geoid in this dual role. In some parts of the world, the elevation data appearing on maps and charts was obtained via this geoid-related approach.

In land areas where heights-above-mean sea level are available from conventional leveling, the geoid, as determined herein, is not needed as the vertical datum (reference surface) for these h-values. Instead, height-above-mean sea level values are determined by conventional leveling from the relationship [6.1]:

$$h = \frac{1}{\bar{g}} \sum_i g_i \Delta h_i \tag{6-3}$$

* When N is negative and the value of h is positive, but smaller than the magnitude of N, the geodetic (ellipsoidal) height of a point is negative. In Florida, for example, the WGS 84 Ellipsoid, if translucent, would be visible above a point (an observer) on the earth's physical surface. (See Figure 6.4.)

where

$\overline{g}$ = mean value of gravity over the plumb line distance between mean sea level and the point where h is required.

g_i = magnitude of gravity measured at leveling site i.

Δh_i = measurement of the difference in elevation between leveling sites i-1 and i.

Due to uncertainties in both H and N, h-values determined by Equation (6-2) are less accurate than those obtained via conventional leveling, Equation (6-3). However, since h-values from conventional leveling are not available for much of the world, Equation (6-2) and the geoid heights central to its use are extremely valuable. Figure 6.5 illustrates h-values determined from the two techniques discussed above. In addition, Figure 6.5 illustrates why geoid heights need not be (and are usually not) zero at coastlines where h = 0. This question is sometimes raised by those unfamiliar with the definition of the geoid as being so many meters above or below the ellipsoid and the fact that the ellipsoid serves to place (position) the geoid. Geoid heights would be zero at the coastlines only if the ellipsoid and geoid intersected (or were tangent) there.

6.2 Formulas and Representations

6.2.1 Formulas

World Geodetic System 1984 Geoid Heights are calculated using a spherical harmonic expansion and the WGS 84 EGM through n=m=180.

The formula for calculating WGS 84 Geoid Heights has the form [6.1]*:

$$N = \frac{GM}{r\gamma} \left[\sum_{n=2}^{n_{max}} \sum_{m=o}^{n} \left(\frac{a}{r}\right)^n (\overline{C}_{n,m} \cos m\lambda + \overline{S}_{n,m} \sin m\lambda) \overline{P}_{n,m}(\sin \phi') \right] \tag{6-4}$$

where

N = Geoid height

GM = The earth's gravitational constant; the product of the universal gravitational constant (G) and the mass of the earth (M). (The latter phrase is only descriptive since the value of GM is determined as a single entity not as a product of the parameters G and M.)

r = Radius vector; radial or geocentric distance to the computation point's location (on the ellipsoid)

γ = Ellipsoidal gravity (at the computation point on the ellipsoid); the value of theoretical (normal) gravity at (on) the surface of the ellipsoid

n,m = Degree and order, respectively, of the spherical harmonic expansion

n_{max} = Maximum degree (and order) of the spherical harmonic expansion

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* Slightly modified. Also, see last paragraph of Section 6.2.2.

a = Semimajor axis of the ellipsoid

$\bar{C}_{n,m}$; $\bar{S}_{n,m}$ = Normalized gravitational coefficients

ϕ' = Geocentric latitude; the angle between the plane of the geodetic equator and the radius vector through the computation point, measured positive north from the equator (0° to 90°), negative south (0° to -90°)

λ = Geocentric longitude (same as geodetic longitude); the angle between the plane of the Zero Meridian, as defined by the BIH, and the plane of the meridian containing the computation point (measured in the plane of the geodetic equator positive eastward from 0° to 360°)

$\bar{P}_{n,m}(\sin \phi')$ = Normalized associated Legendre functions.

In addition, the following mathematical expressions are needed to support the calculation of geoid heights using Equation (6-4):

$$r = a\ (1-e^2)^{1/2} \cdot (1-e^2 \cos^2 \phi')^{-1/2} \qquad (6\text{-}5)$$

e^2 = First eccentricity squared

$e^2 = (a^2-b^2)/a^2$; [numerical value provided below]

b = Semiminor axis of the ellipsoid

$$\tan \phi' = (1-e^2) \tan \phi$$

$$\phi' = \mathrm{Arctan}\ [(1-e^2) \tan \phi] \qquad (6\text{-}6)$$

ϕ = Geodetic latitude; the angle between the plane of the geodetic equator and the normal to the ellipsoid through the computation point, measured positive north from the geodetic equator (0° to 90°), negative south (0° to -90°).

$$\gamma = \gamma_e \frac{1+k \sin \phi^2}{(1-e^2 \sin^2 \phi)^{1/2}} \tag{6-7}$$

γ = Theoretical gravity calculated using the WGS 84 Ellipsoidal Gravity Formula

γ_e = Equatorial gravity; value of theoretical (normal) gravity on the surface of the ellipsoid at the geodetic equator (ϕ=0°); [numerical value provided below in equation for γ, Section 6.2.2]

k = A constant; [numerical value provided below in equation for γ, Section 6.2.2].

For m=o:

$$\bar{P}_{n,o} (\sin \phi') = (2n + 1)^{1/2}\, P_{n,o}(\sin \phi') \tag{6-8}$$

$P_{n,o}(\sin \phi')$ = Legendre Polynomial

$$= \frac{1}{2^n n!} \frac{d^n (\sin^2 \phi' - 1)^n}{d(\sin \phi')^n} . \tag{6-9}$$

For m≠o:

$$\bar{P}_{n,m}(\sin \phi') = \left[\frac{(n-m)!\ (2n+1)2}{(n+m)!} \right]^{1/2} P_{n,m}(\sin \phi') \tag{6-10}$$

$P_{n,m}(\sin \phi')$ = Associated Legendre function

$$= (\cos\phi')^m \frac{d^m P_n(\sin\phi')}{d(\sin\phi')^m}\,. \tag{6-11}$$

6.2.2 Input Data

The following WGS 84 numerical data is needed to proceed with the calculation of WGS 84 Geoid Heights utilizing the above formulas:

$GM = 3986005 \times 10^8\ m^3\ s^{-2}$ (includes the mass of the earth's atmosphere)

$$\gamma = (9.7803267714)\ \frac{1+0.00193185138639\ \sin^2\phi}{(1-0.00669437999013\ \sin^2\phi)^{1/2}}\ m\ s^{-2}$$

$n_{max} = 180$

m = meters

s = seconds

$a = 6378137$ m

$e^2 = 0.00669437999013$

$\bar{C}_{n,m}$, $\bar{S}_{n,m}$ = A total of 32755 gravitational coefficients. These coefficients, when supplemented with GM, comprise the WGS 84 Earth Gravitational Model (EGM). The WGS 84 EGM is classified CONFIDENTIAL with the exception of the coefficients through n=m=18 which are UNCLASSIFIED. NOTE: For calculating WGS 84 Geoid Heights, the $\bar{C}_{2,0}$, $\bar{C}_{4,0}$, $\bar{C}_{6,0}$, $\bar{C}_{8,0}$, and $\bar{C}_{10,0}$ coefficients must be replaced by the quantities:

$\delta\bar{C}_{2,0} = 0.00000000$

$\delta\bar{C}_{4,0} = -0.25330818 \times 10^{-6}$

$\delta\bar{C}_{6,0} = -0.14896096 \times 10^{-6}$

$$\delta\overline{C}_{8,0} = 0.42976374 \times 10^{-7}$$

$$\delta\overline{C}_{10,0} = 0.50931578 \times 10^{-7}.$$

The five preceding coefficient differences were formed using the relationship

$$\overline{\delta C}_{2n,0} = \overline{C}_{2n,0}\text{ (Dynamic)} - \overline{C}_{2n,0}\text{ ("Geometric")}$$

where

$\overline{C}_{2n,0}$ (Dynamic) = WGS 84 EGM coefficients

$\overline{C}_{2n,0}$ ("Geometric") = Computed using in Equation (3-62) the defining parameter $\overline{C}_{2,0}$ and e^2 of the WGS 84 Ellipsoid. (These "geometrically-determined" even degree zonal gravitational coefficients are insignificantly small for other than the five coefficients computed.)

6.2.3 Representations

The geoid is usually depicted as a contour chart which shows the deviations of the geoid from the ellipsoid selected as the mathematical figure of the earth. A worldwide WGS 84 Geoid Height Contour Chart was developed using in Equations (6-4) to (6-11) the above WGS 84 numerical data and the WGS 84 EGM coefficients through n=m=180. A worldwide 1°x1° grid of WGS 84 Geoid Heights was calculated and then contoured using a five meter contour interval to form the chart. This contour chart and other WGS 84-related geoid height graphics are contained in [6.2]. A 10°x10° grid of WGS 84 Geoid Heights (n=m=180) is provided in [6.3] [6.4] along with a contour chart developed by contouring at a five meter contour interval a worldwide 1°x1° grid of WGS 84 Geoid Heights truncated at (n=m=18).

WGS 84 Geoid Heights are available to DoD requesters in representations other than contour charts of various contour intervals and scales. WGS 84 Geoid Heights can be provided on magnetic tape for a specified grid interval or for sites of interest. A Geoid Height Computer Program (n=m=180) capable of calculating both gridded and random point geoid heights can also be made available (along with appropriate documentation and test cases). (See Section 6.5.)

6.3 Geoid Height Interpolation Method

A basic objective with respect to WGS 84 Geoid Heights is that they be based on a utilization of the complete n=m=180 WGS 84 EGM. Adherence to this objective is necessary to ensure that WGS 84 Geoid Heights calculated by different organizations for the same locations are in agreement. For some situations or applications, should it be impractical from either a computer storage or computer time standpoint to calculate geoid heights using the n=m=180 expansion, adherence to the desired expansion can be maintained by interpolating from a pre-computed grid of geoid heights calculated using the n=m=180 WGS 84 EGM. Such an interpolation can be conducted with minimal (and acceptable) accuracy degradation by choosing a small enough grid interval. Use of a 30'x30' grid of geoid heights will provide interpolated values of sufficient accuracy for most applications.

A brief investigation of various grid sizes and geoid height interpolation methods was conducted before coming to the above conclusion. The criteria for selecting a geoid height interpolation method are that it be economical to use, easy to implement, provide good accuracy, and produce consistent values along the grid boundaries. The investigation of interpolation techniques involved two methods, a Weighting Function Method [6.5] and a Bi-Linear Interpolation Method [6.6]. Since the Bi-Linear Interpolation Method provided interpolated values that were more accurate than those obtained using the Weighting Function Method, and adequately met all the criteria for selecting an interpolation method, the Bi-Linear Interpolation Method was selected as the geoid height interpolation scheme to use with a gridded set of WGS 84 Geoid Heights.

The Geoid Height Bi-Linear Interpolation Method has the form

$$N(\phi, \lambda) = a_0 + a_1 X + a_2 Y + a_3 XY \tag{6-12}$$

where

$$a_0 = N_1 \tag{6-13}$$

$$a_1 = N_2 - N_1 \tag{6-14}$$

$$a_2 = N_4 - N_1 \tag{6-15}$$

$$a_3 = N_1 + N_3 - N_2 - N_4 \tag{6-16}$$

$$X = (\lambda - \lambda_1)/(\lambda_2 - \lambda_1) \tag{6-17}$$

$$Y = (\phi - \phi_1)/(\phi_2 - \phi_1) \tag{6-18}$$

Information is provided in Figure 6.6 on the coordinate system associated with the Geoid Height Bi-Linear Interpolation Method.

Accuracy investigations of the Geoid Height Bi-Linear Interpolation Method were initiated by computing WGS 84 Geoid Heights on a worldwide 30'x30' grid (259,200 values), and at the center point of each 30'x30' quadrangle (259,200 values), to serve as truth data. From this data set, "true" WGS 84 Geoid Heights were selected for the four corners of each 1°x1° quadrangle and then used with the Bi-Linear Interpolation Method to interpolate geoid heights at the center point of each 30'x30' cell within the 1°x1° quadrangle. The center point geoid heights (259,200 values), each interpolated using four 1°x1° corner values, were then compared with the "true", previously computed, center point, WGS 84 Geoid Heights. The root-mean-square (RMS) of the differences was ±0.25 meter, with the largest positive and negative differences being 4.16 and -3.53 meters, respectively (Table 6.1). Next, the four "true" WGS 84 Geoid Heights previously computed for the corners of each 30'x30' quadrangle were used with the Bi-Linear Interpolation Method to also interpolate geoid heights at the center point of each 30'x30' quadrangle. As before, the interpolated center point geoid heights (259,200 values) were compared with the previously computed, similarly located, "true" WGS 84 Geoid Heights. The RMS of the differences was ±0.09 meter, with the largest positive and negative differences being 1.55 and -1.34 meters, respectively (Table 6.2). From Table 6.2, it is also noted that only 32 of these geoid height differences are larger than 1 meter. Table 6.3 and Figure 6.7 show their geographical distribution. Due to the location and small number of interpolated geoid heights (32) that differ from their true values by more than a meter, it was concluded that a 30'x30' grid of geoid heights and the Bi-Linear Interpolation Method can be used to interpolate WGS 84 Geoid Heights to an interpolation error acceptable for most applications when the slightly more accurate direct calculation of geoid heights using Equation (6-4) is not practical.

6.4 Analysis/Accuracy of WGS 84 Geoid

A worldwide 1° x 1° grid of WGS 84 Geoid Heights was computed and compared with a similar grid of WGS 72 Geoid Heights referenced to the WGS 72 Ellipsoid. A contour chart of these geoid height differences is available in [6.2]. The RMS difference was ±4.6 meters, with the largest positive and negative differences being 24 and -23.5 meters, respectively. These and additional statistics from the comparison are listed in Table 6.4. Note from Table 6.4 that 13,841 of the 64,800 geoid height differences, or 21.36 percent of the differences, are larger then 5 meters.

The RMS WGS 84 Geoid Height, taken worldwide on the basis of a 1° x 1° grid, is 30.5 meters. This RMS value indicates how well the WGS 84 Ellipsoid, taken as the mathematical figure of the earth, fits the earth's mean sea level surface.

It is generally acknowledged that the most accurate geoid heights available today from an absolute accuracy standpoint are satellite-derived geometric geoid heights. These geoid heights are obtained by expressing Equation (6-1) in the form

$$N = H - h \quad . \tag{6-19}$$

In the context of this discussion:

N = satellite-derived geometric geoid height
H = satellite-derived geodetic height
h = height-above-mean sea level value obtained via conventional leveling.

From the error relationship

$$\varepsilon_N = \varepsilon_H - \varepsilon_h \tag{6-20}$$

and the lack of correlation between the errors (ε) in H and h, assumed random:

$$\sigma_N^2 = \sigma_H^2 + \sigma_h^2$$

$$\sigma_N = (\sigma_H^2 + \sigma_h^2)^{1/2} \quad . \tag{6-21}$$

If H is laser derived, σ_H is approximately ±1 meter. For Doppler-derived WGS 84 H-values, determined via satellite point positioning, σ_H ranges from ±1 to ±4 meters. See Equations (9-2), (9-4), and (9-6). The absolute error in h (σ_h) at well-surveyed satellite tracking sites is on the order of ±1 to ±2 meters. Using these values for σ_H and σ_h in Equation (6-21), and rounding to the nearest meter:

$$\sigma_N = \pm 1 \text{ to } \pm 4 \text{ meters.} \tag{6-22}$$

Therefore, Equation (6-22) establishes ±1 meter (one sigma) as the current absolute accuracy threshold, at well-surveyed tracking stations, for satellite-derived geometric geoid heights (relative to the WGS 84 Ellipsoid).

The preceding accuracy information was utilized through geoid height comparisons, Doppler-derived and laser-derived WGS 84 geometric geoid heights versus WGS 84 (n=m=180) Geoid Heights, to establish the accuracy threshold for WGS 84 Geoid Heights [6.2]. The WGS 84 Geoid Heights have an error range of ±2 to ±6 meters (one sigma), and are known to accuracies of ±2 to ±3 meters over approximately 55 percent of the earth. Approximately 93 percent of the earth has WGS 84 Geoid Heights of accuracy better than ±4 meters. A worldwide WGS 84 Geoid Height Accuracy Graphic and error analysis particulars are available in [6.2].

6.5 Availability of WGS 84 Geoid Height Data

WGS 84 Geoid Height data and products that can be provided to requesters include:

- A worldwide WGS 84 Geoid Height Contour Chart (developed by contouring a 1°x1° grid of values), contour interval = 5 meters. If needed, contour charts of various physical size can be provided based on other contour intervals and scales, and for specific geographic areas.

- A magnetic tape containing the worldwide 1°x1° grid of WGS 84 Geoid Heights used in developing the worldwide WGS 84 Geoid Height Contour Chart.

- A magnetic tape containing a worldwide 30'x30' grid of WGS 84 Geoid Heights, plus a Bi-Linear Interpolation Method for interpolating WGS 84 Geoid Heights at random points. As stated previously, this interpolation scheme has an interpolation error (RMS difference) of ± 0.09 meter based on a comparison between 259,200 true and interpolated WGS 84

Geoid Heights. Only 32 geoid height differences exceeded 1 meter, the largest difference being 1.55 meters.

- A Computer Program for calculating WGS 84 Geoid Heights at a specified grid interval or at random points. Associated documentation and appropriate test cases are included.

Requests for WGS 84 Geoid Height data and products should be forwarded to the address provided in the PREFACE.

6.6 Comments

Due to the absence of tide gauges and other related problems, much of the world today does not have an accurate vertical datum (reference) for elevation data. Also, those vertical datums that do exist are not consistent with one another. Much has been written in the last few years about the need for a new accurate worldwide vertical datum. Such a datum can easily be established by using the WGS 84 Geoid to provide coastline values (geoid heights) at sites (h=0) from which conventional leveling can proceed and, where such leveling is absent, elevation data can be established from Equation (6-2) where such h-values are automatically related to the WGS 84 Geoid and the WGS 84 Ellipsoid. With this approach, the WGS 84 Ellipsoid can serve as the reference for elevation data (as it now serves as the reference for horizontal positions), while at the same time the concept of heights-above-mean sea level is retained. Although in the literature some researchers propose abandoning use of the geoid, it is important for some vertical positioning applications that mean sea level be retained at this time. For example, aircraft equipped with barometric altimeters measure their height above mean sea level.

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Table 6.1

WGS 84 Geoid Height Differences
- True (Calculated) Values Minus Values Interpolated from a 1°x1° Grid -

Range of Geoid Height Differences (Meters)	Frequency of Differences	
	Positive Differences	Negative Differences
$0.0 \leq 0.5$	120,766	126,741
$> 0.5 \leq 1.0$	5,043	4,106
$> 1.0 \leq 1.5$	864	872
$> 1.5 \leq 2.0$	217	363
$> 2.0 \leq 2.5$	60	98
$> 2.5 \leq 3.0$	6	44
$> 3.0 \leq 3.5$	0	15
> 3.5	4	1
Largest Difference	4.16 m	-3.53 m
RMS Difference	±0.25 m	
Mean Difference	0.00002 m	
Number of Differences	259,200	

Table 6.2

WGS 84 Geoid Height Differences
- True (Calculated) Values Minus Values Interpolated from a 30'x30' Grid -

Range of Geoid Height Differences (Meters)	Frequency of Differences	
	Positive Differences	Negative Differences
0.0 ≤ 0.2	122,216	127,215
> 0.2 ≤ 0.4	4,265	3,525
> 0.4 ≤ 0.6	644	812
> 0.6 ≤ 0.8	174	227
> 0.8 ≤ 1.0	32	58
> 1.0 ≤ 1.2	4	23
> 1.2 ≤ 1.4	0	3
> 1.4 ≤ 1.6	2	0
Largest Difference	1.55 m	-1.34 m
RMS Difference	±0.09 m	
Mean Difference	0.000001 m	
Number of Differences	259,200	

Table 6.3

Distribution of Geoid Height Differences Greater Than One Meter
- True (Calculated) Values Minus Values Interpolated from a 30' x 30' Grid* -

Geodetic Latitude

+90°

2 Points > | 1.0 |
[-1.04 -1.00]

+45

25 Points > | 1.0 |
[-1.23 1.55]

0

5 Points > | 1.0 |
[-1.34 -1.00]

-45

0 Points > | 1.0 |

-90°

0° Geodetic Longitude 360°

*259,200 Values Compared.

Table 6.4

Geoid Height Differences
- WGS 84 Minus WGS 72* -

Range of Geoid Height Differences (Meters)	Frequency of Occurrence	Percent of Total Values (64,800)
0 - 1	13096	20.21
> 1 ≤ 2	12038	18.58
> 2 ≤ 3	10585	16.33
> 3 ≤ 4	8591	13.26
> 4 ≤ 5	6649	10.26
> 5 ≤ 10	12163	18.77
> 10 ≤ 15	1484	2.29
> 15 ≤ 20	182	0.28
> 20	12	0.02
0 - 2	25134	38.79
0 - 3	35719	55.12
0 - 4	44310	68.38
0 - 5	50959	78.64
0 - 10	63122	97.41
0 - 15	64606	99.70
0 - 20	64788	99.98

Largest Positive Difference = 24.0 m	RMS Difference = ±4.6 m
Largest Negative Difference = -23.5 m	Mean Difference = 0.0001 m

*WGS 72 Geoid Heights Referenced to WGS 72 Ellipsoid

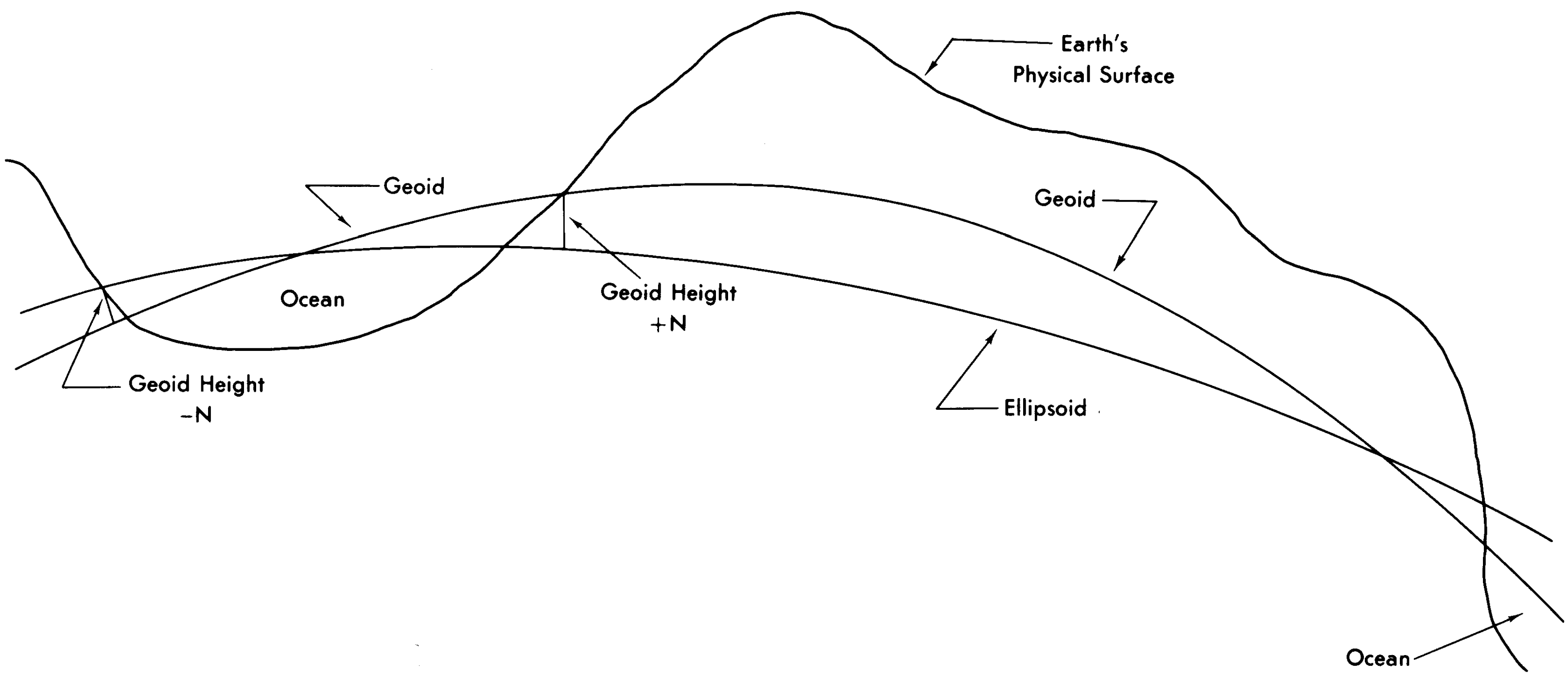


Figure 6.1. Relationship Between the Earth's Physical Surface, the Mean Sea Level Surface of the Earth (or Geoid), and the Earth's Mathematical Figure (an Ellipsoid)

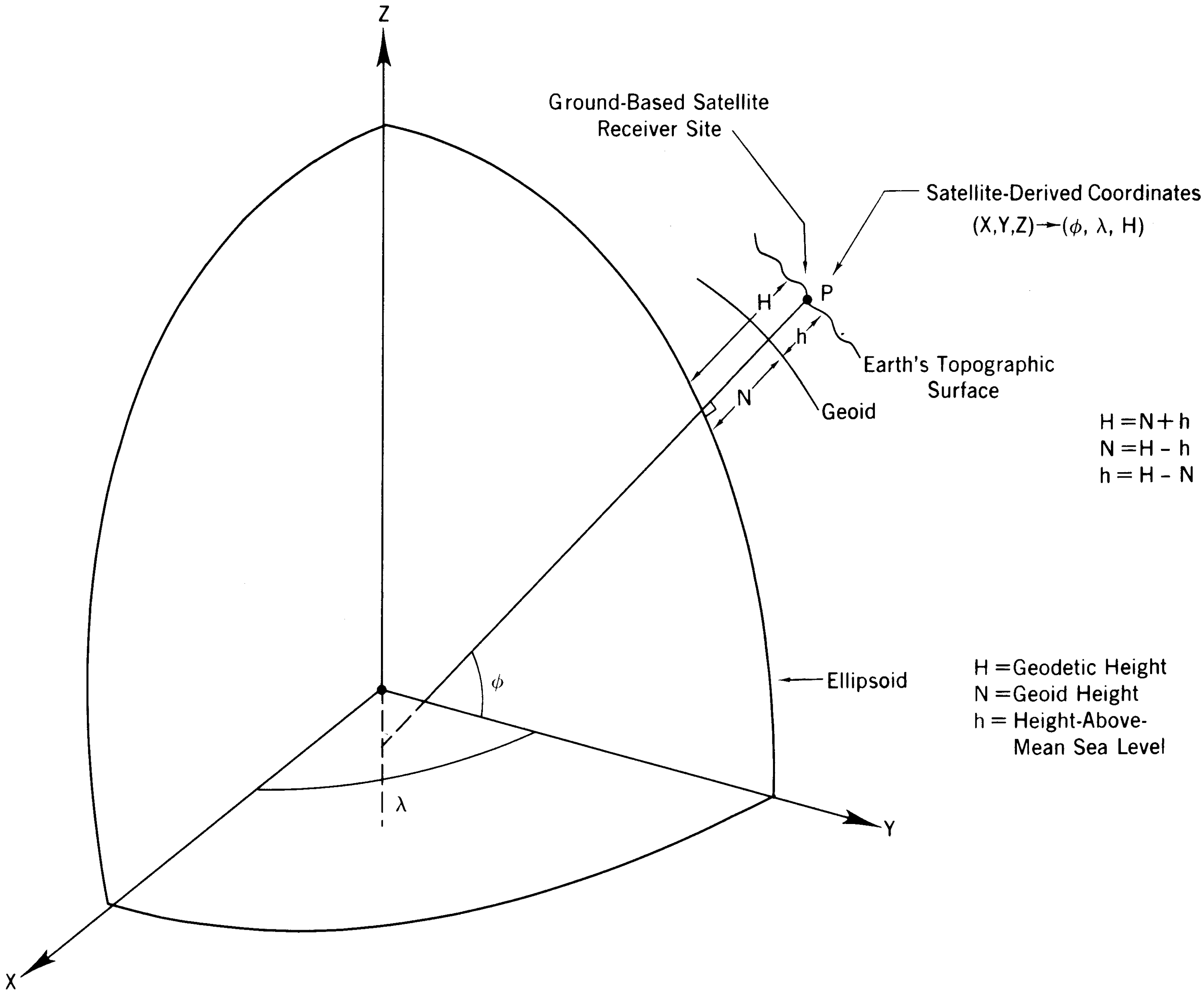


Figure 6.2. Satellite-Derived Height-Above-Mean Sea Level Values (Terrestrial Points)

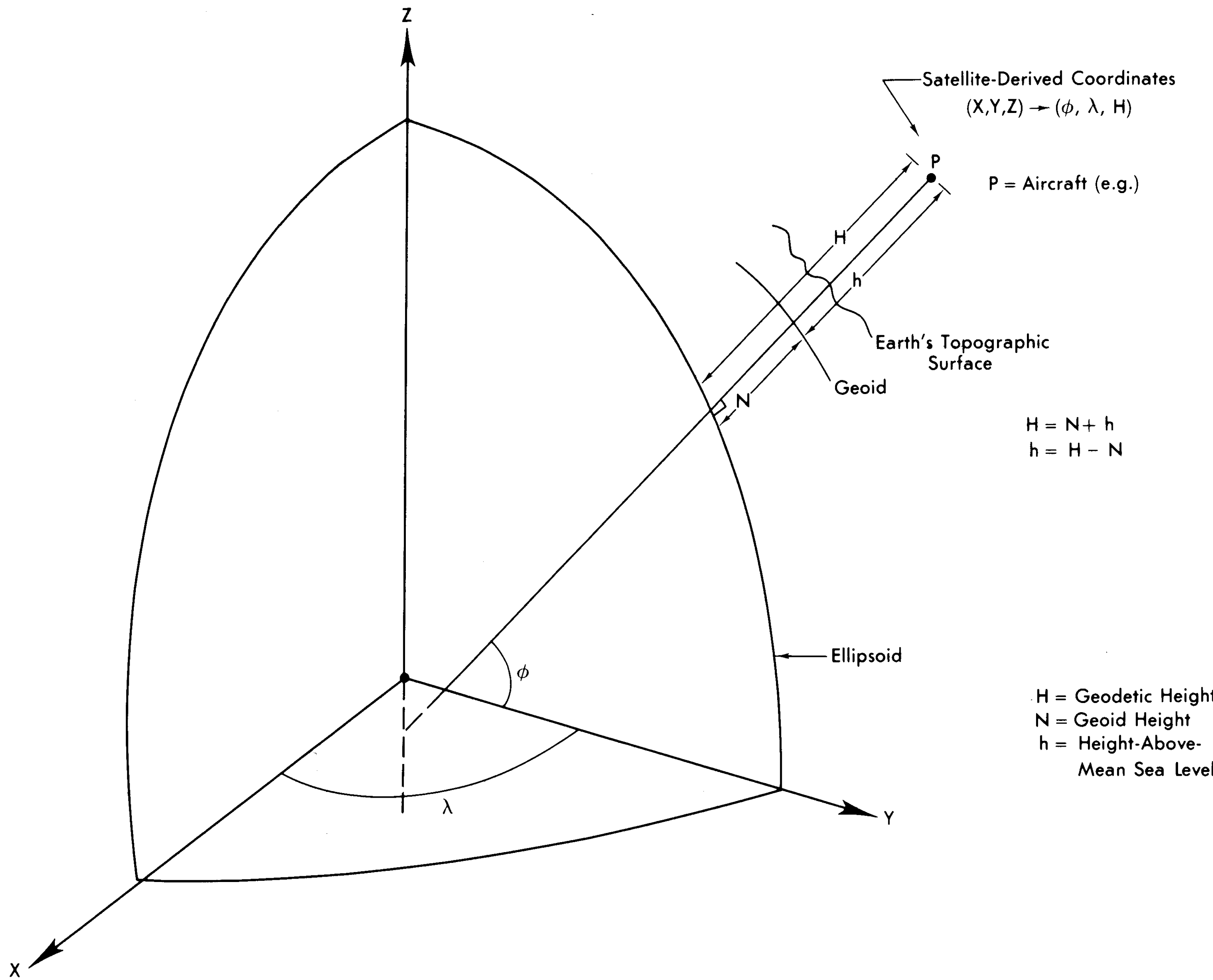


Figure 6.3. Satellite-Derived Height-Above-Mean Sea Level Values (Spatial Objects)

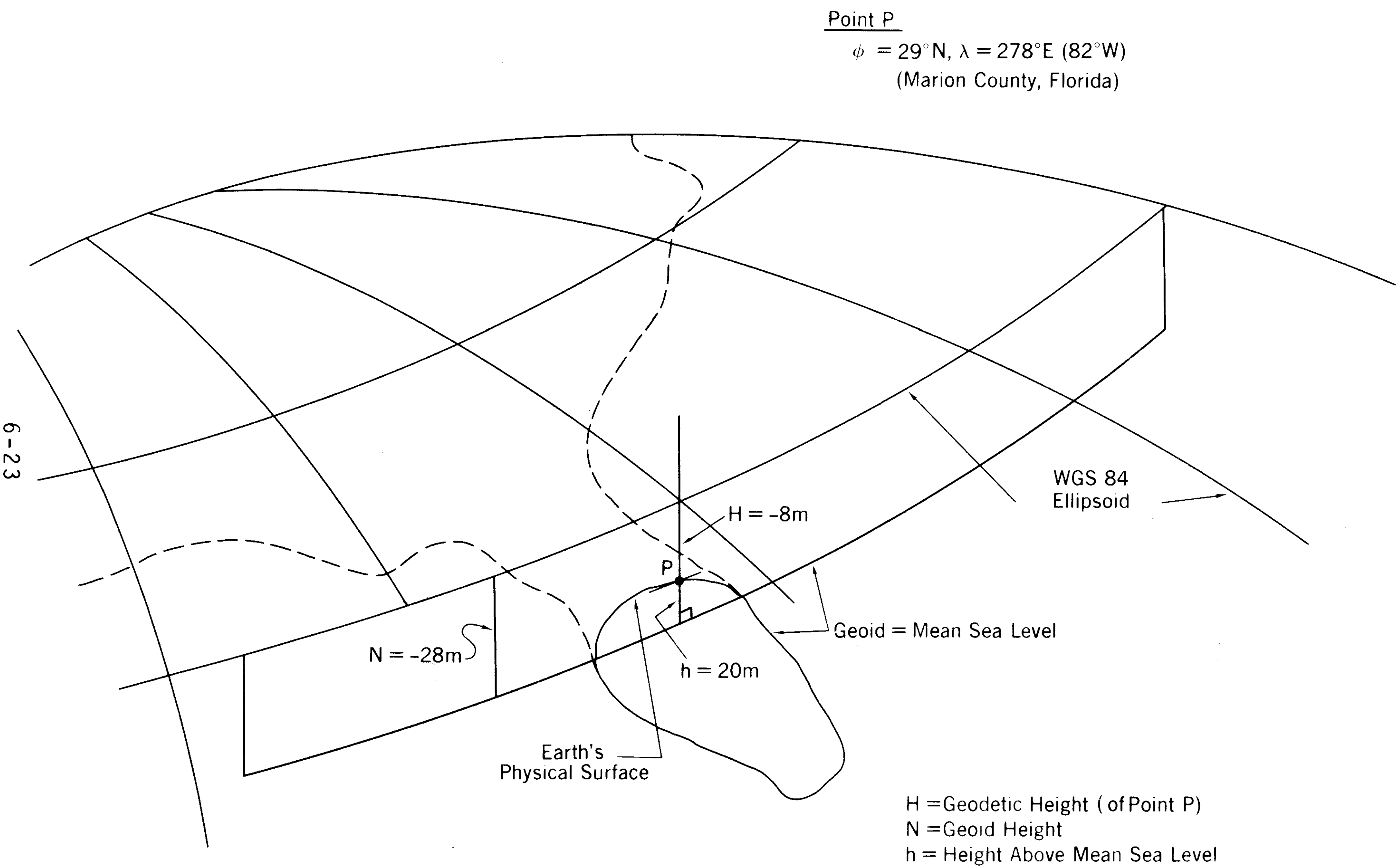
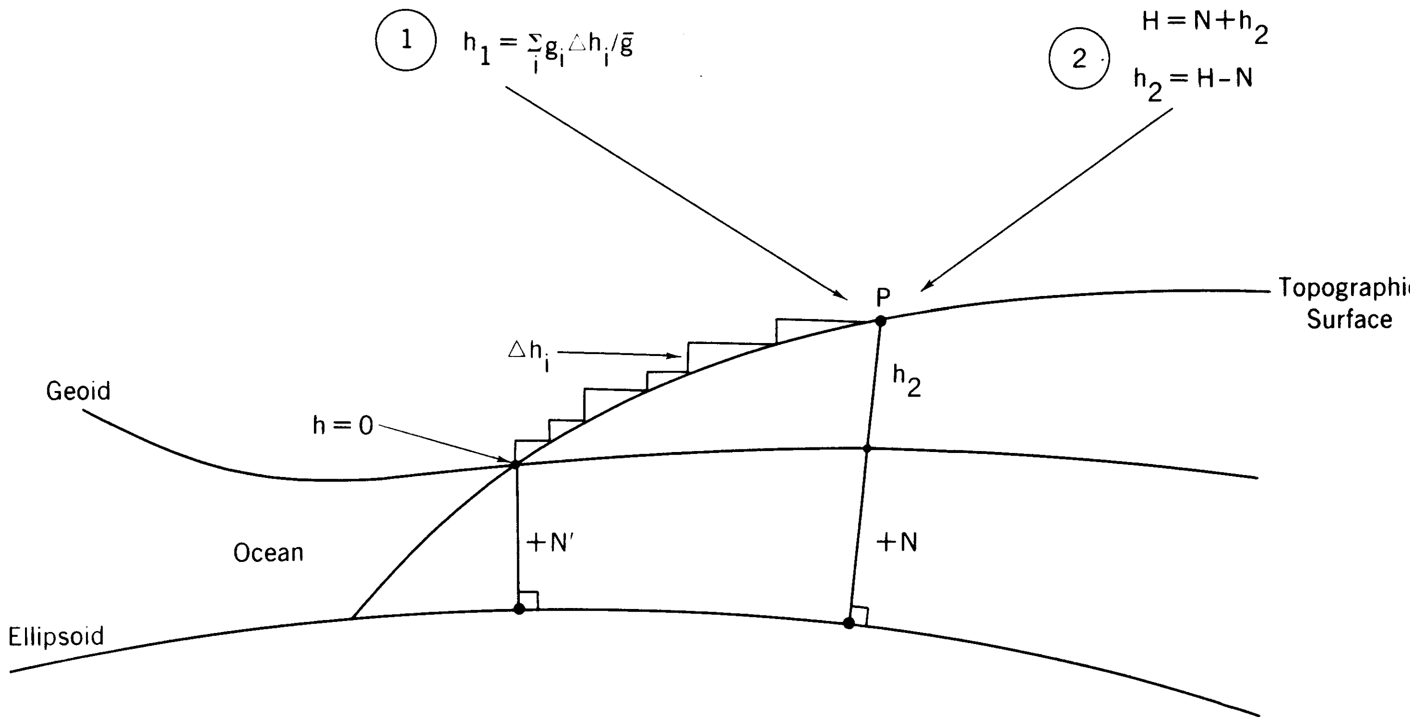


Figure 6.4. Illustration of a Negative WGS 84 Geodetic Height (H)



$$h_1(P) \neq h_2(P)$$

$$\sigma_{h_1}(P) < \sigma_{h_2}(P)$$

$$\sigma_{h_1}, \sigma_{h_2} = \text{Errors in } h_1,\ h_2 \text{ at P}$$

Figure 6.5. Two Techniques for Obtaining Height Above Mean Sea Level Values

$N_P(\phi_P, \lambda_P)$ = Geoid Height to be Interpolated at Point P

N_1, N_2, N_3, N_4 = Available Grid Point Geoid Heights

X,Y = Non-Dimensional Coordinates of Point P

$$X = (\lambda_P - \lambda_1)/(\lambda_2 - \lambda_1)$$
$$Y = (\phi_P - \phi_1)/(\phi_2 - \phi_1)$$

[Equations (6.17), (6.18)]

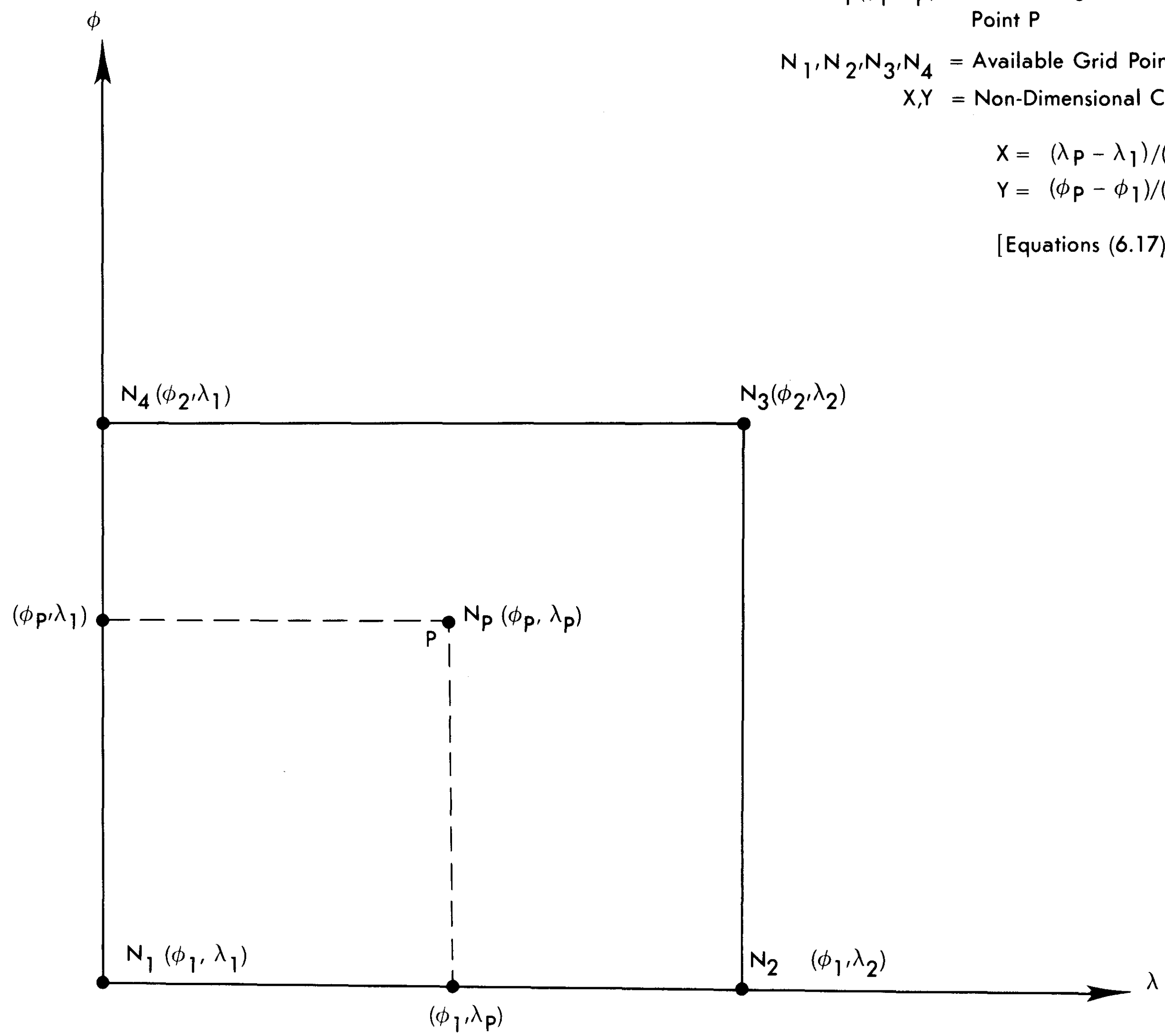


Figure 6.6. Coordinate System Associated With Geoid Height Bi-Linear Interpolation Scheme

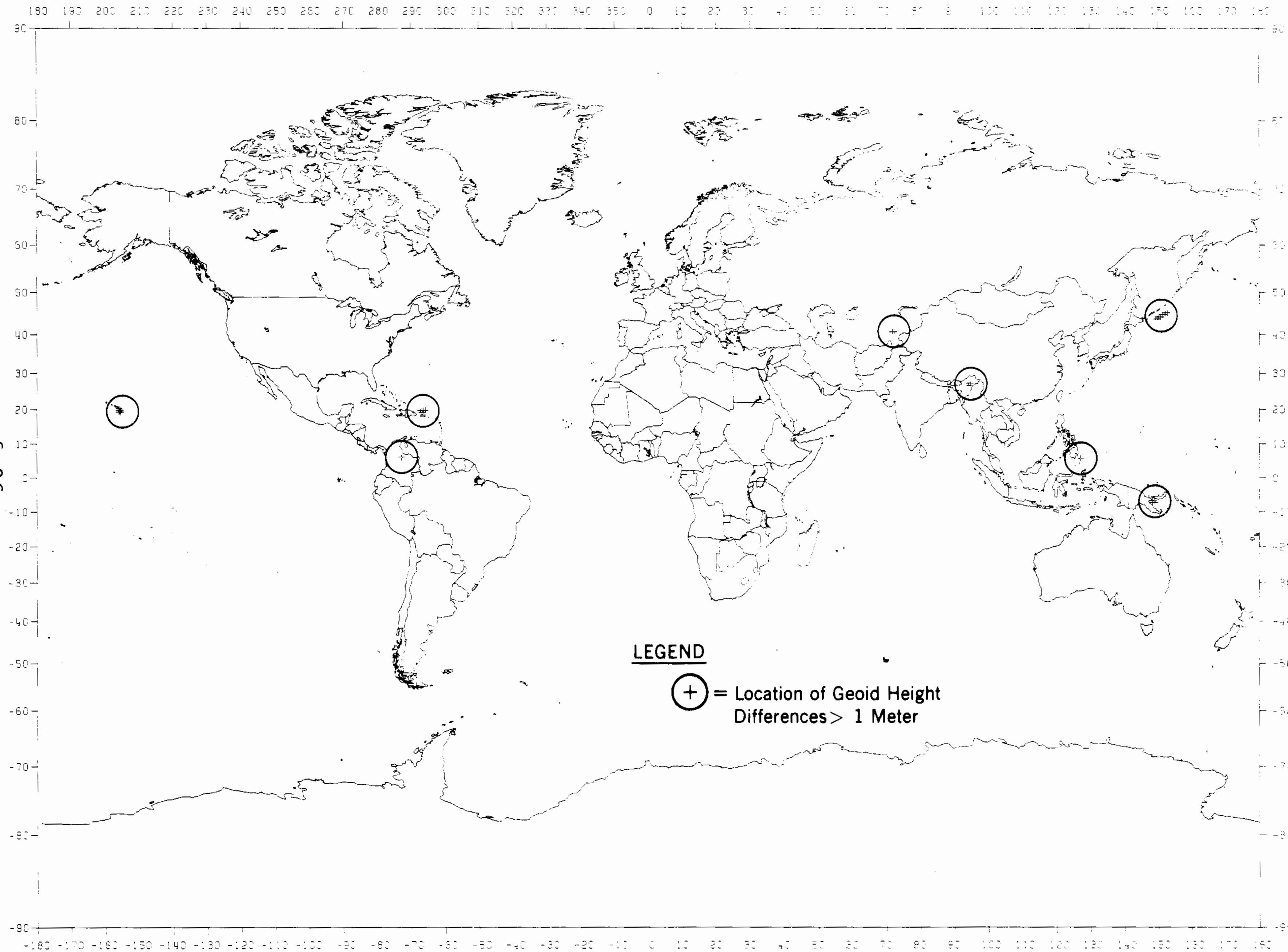


Figure 6.7. Location of Geoid Height Differences Greater Than 1 Meter (True Minus Interpolated From a 30'x30' Grid)