4. WGS 84 ELLIPSOIDAL GRAVITY FORMULA

4.1 General

In Section 3.1, the WGS 84 Ellipsoid is identified as being a geocentric equipotential ellipsoid of revolution. An equipotential ellipsoid is simply an ellipsoid defined to be an equipotential surface, i.e., a surface on which all values of the gravity potential are equal. Given an ellipsoid of revolution, it can be made an equipotential surface of a certain potential function, the theoretical (normal) gravity potential (U). This theoretical gravity potential can be uniquely determined, independent of the density distribution within the ellipsoid, by using any system of four independent constants as the defining parameters of the ellipsoid. As noted earlier for the WGS 84 Ellipsoid (Chapter 3), these are the semimajor axis (a), the normalized second degree zonal gravitational coefficient ($\bar{C}_{2,0}$), the earth's angular velocity (ω), and the earth's gravitational constant (GM).

To determine the theoretical gravity potential without resorting to the use of a mass distribution model for the ellipsoid, U can be expanded into a series of zonal ellipsoidal harmonics of linear eccentricity in $(a^2 - b^2)^{1/2}$. The coefficients in the series are determined by using the condition that the ellipsoid (Chapter 3) is an equipotential surface

$$U = U_0 = \text{Constant} . \qquad (4\text{-}1)$$

Since all the zonal ellipsoidal harmonic coefficients vanish, except the two of degree zero and two, a closed finite expression is obtained for U [4.1; pp 64-66].

Theoretical gravity (γ) , the gradient of U, is given on (at) the surface of the ellipsoid by the closed formula of Somigliana [4.1; p 70]:

$$\gamma = (a\ \gamma_e \cos^2\phi + b\ \gamma_p \sin^2\phi)/(a^2\cos^2\phi + b^2\sin^2\phi)^{1/2} \qquad (4\text{-}2)$$

where

a, b = semimajor and semiminor axes of the ellipsoid, respectively

γ_e, γ_p = theoretical gravity at the equator and poles, respectively

ϕ = geodetic latitude.

Thus, the equipotential ellipsoid serves not only as the reference surface or geometric figure of the earth, but leads to a closed formula for theoretical gravity at the ellipsoidal surface.

4.2 Analytical and Numerical Forms

4.2.1 Formulas

The closed gravity formula of Somigliana in the form [4.2]

$$\gamma = \gamma_e\ (1 + k\ \sin^2\phi)/(1 - e^2\sin^2\phi)^{1/2} \qquad (4\text{-}3)$$

has been selected as the official WGS 84 Ellipsoidal Gravity Formula. In Equation (4-3):

$$k = (b\ \gamma_p / a\ \gamma_e) - 1 \qquad (4\text{-}4)$$

e^2 = square of the first eccentricity of the ellipsoid.

Equation (4-3) was selected for use with WGS 84 in preference to Equation (4-2) since it is more convenient for numerical computations and explicitly contains γ_e as the first factor in the equation.

The WGS 84 Ellipsoidal Gravity Formula, expressed numerically, is

$$\gamma = 9.7803267714 \frac{1 + 0.00193185138639 \sin^2\phi}{(1 - 0.00669437999013 \sin^2\phi)^{1/2}} \text{ m s}^{-2} \qquad (4\text{-}5)$$

where

$\gamma_e = 9.7803267714$ m s^{-2}

or

$\gamma_e = 978.03267714$ cm s^{-2} (Gals)

$\gamma_e = 978032.67714$ milligals.

In the preceding:

1 Gal = an acceleration due to gravity of 1 centimeter/second squared

1 milligal = an acceleration due to gravity of 1 x 10^{-3} centimeters/second squared.

4.2.2 Derivation

Several geometric and physical constants derived from the defining parameters of the WGS 84 Ellipsoid are needed in transforming the analytical expression for the WGS 84 Ellipsoidal Gravity Formula, Equation (4-3), to numerical form, Equation (4-5). The fundamental derived geometric constant is e^2, which is related to the four defining parameters of the WGS 84 Ellipsoid via Equation (3-24). Equation (3-24), repeated here for convenience,

$$e^2 = -3\,(5^{1/2})\,\mathcal{C}_{2,0} + \frac{4}{15}\,\frac{\omega^2 a^3}{GM}\,\frac{e^3}{2q_o} \tag{4-6}$$

is solved iteratively for e^2, taking into account

$$2q_o = (1 + \frac{3}{e'^2})\,(\arctan e') - \frac{3}{e'} \tag{4-7}$$

and the following expression for the second eccentricity (e'):

$$e' = e(1-e^2)^{-1/2}\ . \tag{4-8}$$

With the derived geometric constant e^2 available, another needed geometric constant, the semiminor axis of the ellipsoid (b), can be derived using the expression [4.2]:

$$b = a\,(1 - e^2)^{1/2}\ . \tag{4-9}$$

By visually inspecting Equations (4-3) and (4-4), it is apparent that the derived physical constants needed to complete the numerical development of the WGS 84 Ellipsoidal Gravity Formula are γ_e and γ_p, the values of theoretical gravity on (at) the surface of the ellipsoid at the equator and the poles, respectively. Knowing the defining parameters of the WGS 84 Ellipsoid, and with the derived geometric constants e^2, e', and b available, Equations (3-65), (3-66), and the following Equation (4-10) can be used to compute q_o', q_o, and m, respectively:

$$m = \omega^2 a^2 b/GM\ . \tag{4-10}$$

These results are then used in Equations (3-63) and (3-64) to obtain numerical values for γ_e and γ_p.

The analytical and numerical forms of the WGS 84 Ellipsoidal Gravity Formula are provided in Table 4.1. The WGS 84

Ellipsoidal Gravity Formula, numerical form, and the WGS 84 Ellipsoid-related defining and derived parameters used in its determination are provided in Table 4.2. For user convenience, values of theoretical gravity on the surface of the WGS 84 Ellipsoid are provided in Table 4.3 at 1° intervals of geodetic latitude. These values, in milligal (mgal) units, were computed using Equation (4-5).

4.3 Average Value of Theoretical Gravity ($\bar{\gamma}$)

4.3.1 Average Value (WGS 84 Ellipsoid)

A number of formulas used in gravimetric geodesy applications require an average value of theoretical gravity. The average value of theoretical gravity for the WGS 84 Ellipsoid is

$$\bar{\gamma} = 9.7976446561 \text{ m s}^{-2}$$

$$\bar{\gamma} = 979764.46561 \text{ milligals.} \qquad (4\text{-}11)$$

(Although many texts represent the average value of theoretical gravity by G, that symbol is not used here since it is already used to denote the universal gravitational constant and also appears in GM.)

4.3.2 Derivation of $\bar{\gamma}$ (WGS 84 Ellipsoid)

The general equation for calculating the average value of theoretical gravity for the earth is

$$\bar{\gamma} = \iint_S \gamma ds \Big/ \iint_S ds \qquad (4\text{-}12)$$

where

γ = value of theoretical gravity
S = surface of the earth
ds = surface element.

For a rotational ellipsoid, Equation (4-12) becomes

$$\bar{\gamma} = \int_{\lambda}\int_{\phi} \gamma\, R_M R_N \cos\phi \, d\phi \, d\lambda \Big/ \int_{\lambda}\int_{\phi} R_M R_N \cos\phi \, d\phi \, d\lambda \qquad (4\text{-}13)$$

where

R_M = radius of curvature in the meridian

$$R_M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2\phi)^{3/2}} \qquad (4\text{-}14)$$

R_N = radius of curvature in the prime vertical

$$R_N = \frac{a}{(1 - e^2 \sin^2\phi)^{1/2}} \qquad (4\text{-}15)$$

a = semimajor axis

e = first eccentricity.

Upon inserting the expressions for R_M and R_N into Equation (4-13), considering that neither they nor theoretical gravity (γ) are a function of longitude, and considering the symmetry of these functions with respect to the equator, the expression for $\bar{\gamma}$ becomes [4.2]:

$$\bar{\gamma} = \int_0^{\pi/2} \frac{\gamma \cos\phi \, d\phi}{(1 - e^2 \sin^2\phi)^2} \Big/ \int_0^{\pi/2} \frac{\cos\phi \, d\phi}{(1 - e^2 \sin^2\phi)^2} \,. \qquad (4\text{-}16)$$

In order to evaluate this equation, both γ and $(1 - e^2 \sin^2\phi)^{-2}$ are expanded in a series and the resultant equation integrated term by term.

Inspecting the denominator of Equation (4-16),

$$\int_0^{\pi/2} \frac{\cos\phi \, d\phi}{(1 - e^2 \sin^2\phi)^2} = \int_0^{\pi/2} (1 - e^2 \sin^2\phi)^{-2} \cos\phi \, d\phi \,, \qquad (4\text{-}17)$$

it is noted that a series expansion is needed for the expression $(1 - e^2 \sin^2\phi)^{-2}$. In series form:

$$(1 - e^2\sin^2\phi)^{-2} = 1 + 2e^2\sin^2\phi + 3e^4\sin^4\phi + 4e^6\sin^6\phi + 5e^8\sin^8\phi + \ldots \tag{4-18}$$

Inserting Equation (4-18) into (4-17) and performing the integration:

$$\int_0^{\pi/2} (1 - e^2\sin^2\phi)^{-2} \cos\phi \, d\phi = 1 + \frac{2}{3} e^2 + \frac{3}{5} e^4 + \frac{4}{7} e^6 + \frac{5}{9} e^8 + \ldots \tag{4-19}$$

Since this result represents the denominator of the equation for the mean value of theoretical gravity, Equation (4-16), its reciprocal must be determined. The reciprocal is

$$\begin{aligned}[\int_0^{\pi/2} \frac{\cos\phi \, d\phi}{(1 - e^2\sin^2\phi)^2}]^{-1} &= [1 + \frac{2}{3} e^2 + \frac{3}{5} e^4 + \frac{4}{7} e^6 + \frac{5}{9} e^8 + \ldots]^{-1} \\ &= 1 - \frac{2}{3} e^2 - \frac{7}{45} e^4 - \frac{64}{945} e^6 \\ &\quad - \frac{512}{14175} e^8 - \ldots \end{aligned} \tag{4-20}$$

To evaluate the numerator of Equation (4-16),

$$\int_0^{\pi/2} \frac{\gamma \cos\phi \, d\phi}{(1 - e^2 \sin^2\phi)^2} = \int_0^{\pi/2} \gamma \, (1 - e^2 \sin^2\phi)^{-2} \cos\phi \, d\phi \, ,$$

it is necessary to have a series expansion for both theoretical gravity (γ) and $(1 - e^2 \sin^2\phi)^{-2}$. Using the truncated series expansion for theoretical gravity [4.2],

$$\gamma \approx \gamma_e(1 + a_2 \sin^2\phi + a_4 \sin^4\phi + a_6 \sin^6\phi + a_8 \sin^8\phi)$$

and Equation (4-18), leads to

$$\begin{aligned}&\int_0^{\pi/2} \gamma \, (1 - e^2\sin^2\phi)^{-2} \cos\phi \, d\phi \\ &\quad = \int_0^{\pi/2} \gamma_e(1 + a_2\sin^2\phi + a_4\sin^4\phi + a_6\sin^6\phi + a_8\sin^8\phi) \\ &\qquad \cdot (1 + 2e^2\sin^2\phi + 3e^4\sin^4\phi + 4e^6\sin^6\phi + 5e^8\sin^8\phi) \cos\phi \, d\phi \, .\end{aligned}$$

This expression can be written as:

$$\int_0^{\pi/2} \gamma(1 - e^2\sin^2\phi)^{-2} \cos\phi \, d\phi$$

$$= \gamma_e \int_0^{\pi/2} [1 + (\frac{5}{8} e^2 + k) \sin^2\phi + (\frac{35}{8} e^4 + \frac{5}{2} e^2 k) \sin^4\phi$$

$$+ (\frac{105}{16} e^6 + \frac{35}{8} e^4 k) \sin^6\phi + (\frac{1155}{128} e^8 + \frac{105}{16} e^6 k) \sin^8\phi] \cos\phi \, d\phi$$

$$= \gamma_e [1 + (\frac{5}{6} e^2 + \frac{1}{3} k) + (\frac{7}{8} e^4 + \frac{1}{2} e^2 k) \tag{4-21}$$

$$+ (\frac{15}{16} e^6 + \frac{5}{8} e^4 k) + (\frac{385}{384} e^8 + \frac{35}{48} e^6 k)].$$

Combining the expressions for the numerator and denominator, Equations (4-21) and (4-20), respectively, the equation for calculating the mean value of theoretical gravity over a rotational ellipsoid is

$$\bar{\gamma} = \gamma_e(1 + \frac{1}{6} e^2 + \frac{1}{3} k + \frac{59}{360} e^4 + \frac{5}{18} e^2 k + \frac{2371}{15120} e^6 \tag{4-22}$$

$$+ \frac{259}{1080} e^4 k + \frac{270229}{1814400} e^8 + \frac{9623}{45360} e^6 k)$$

where γ_e , e^2, and k are defined as before.

Using Equation (4-22) and values for γ_e, e^2, and k from Table 4.1, the average value of theoretical gravity for the WGS 84 Ellipsoid was computed. This value was provided earlier as Equation (4-11).

4.4 Atmospheric Effects

4.4.1 Theoretical Considerations

In the discussion on the equipotential ellipsoid (Section 3.2), it was stated that the reference ellipsoid "is defined to enclose the whole mass of the Earth, including the atmosphere". This, of course, results from the adoption of a GM value which includes the mass of the atmosphere. As a result, the theoretical gravity formula derived for WGS 84 is for an ellipsoid that includes the mass of the atmosphere. This permits the theoretical gravity field to be computed at the ellipsoid surface and in space without having to consider the variation in atmospheric density.

Use of a GM value, in the development of the theoretical gravity formula, that includes the mass of the atmosphere is a deviation from what was done in the development of previous world geodetic system ellipsoidal gravity formulas. Therefore, caution must be exercised when using the WGS 84 Ellipsoidal Gravity Formula to ensure that it is implemented correctly. For those situations which require that atmospheric effects be considered, this is done by applying corrections to the measured values. Therefore, the burden of taking atmospheric effects into account is transferred from the reference system to the gravity data measurement/reduction process.

4.4.2. Atmospheric Correction to Measured Gravity

In the IAG publication on Geodetic Reference System 1967 [4.3], a detailed derivation is given of the correction to measured gravity for the effect of the earth's atmosphere. The publication also contains a table of atmospheric correction values, δg_A, which are to be added to measured gravity, when gravity anomalies are being formed. Three sets of δg_A values are given in the table:

- Values calculated to 1×10^{-8} m s^{-2} (or 0.001 mgal), using the Committee for Space Research (COSPAR) International Reference Atmosphere (CIRA 1961).

- Values calculated to 1×10^{-8} m s^{-2} (or 0.001 mgal), using the United States Standard Atmosphere.

- The average of the results obtained using the two above atmospheric models, rounded to 1×10^{-7} m s^{-2} (or 0.01 mgal).

The set of average atmospheric corrections was recommended by the IAG for use with both GRS 67 and GRS 80 . For consistency, this set is also recommended for use when forming WGS 84 gravity anomalies. For ease of reference, this set of atmospheric corrections is provided in Table 4.4 for elevations up to 34 kilometers, at 0.5 kilometer elevation increments up to 10 kilometers and at larger increments at higher elevations. These corrections are also depicted graphically in Figure 4.1.

For those applications where use of Table 4.4 is inconvenient or perhaps cumbersome for estimating needed atmospheric correction values, the following empirically derived equation may be used in its place [4.4]:

$$\delta g_A = 0.87\, e^{-0.116\, h^{1.047}} \text{ mgal.} \tag{4-23}$$

In Equation (4-23), which reproduces Table 4.4 to an RMS accuracy of ± 0.0094 mgal, h is the height of the gravity station above mean sea level in kilometers. Atmospheric correction values determined from Equation (4-23) differ from Table 4.4 values by less than 0.01 mgal for elevations up to 10 kilometers, with a maximum difference of 0.0224 mgal, which occurs when h = 15 kilometers.

As stated above, the atmospheric correction values are to be added to measured gravity values when the latter are used along with theoretical gravity (WGS 84 Ellipsoidal Gravity Formula) values to obtain WGS 84 gravity anomalies. The relevant formula is

$$\Delta g_{84} = g + \delta g_A - \gamma_{84} + \text{gravity reduction terms} \quad (4\text{-}24)$$

where

Δg_{84} = gravity anomaly referenced to the WGS 84 Ellipsoid, and of type corresponding to the gravity reduction terms applied

g = value of gravity measured on the earth's physical surface (and referenced to the International Gravity Standardization Net 1971) [4.5]

δg_A = atmospheric correction to measured gravity (at the elevation above mean sea level of the gravity station)

γ_{84} = value of theoretical gravity calculated using the WGS 84 Ellipsoidal Gravity Formula

and "gravity reduction terms" pertain to the type of gravity reduction applied (e.g., free-air, Bouguer, etc.).

4.5 Gravity Anomaly Conversion

4.5.1. Recommended Approach

When implementing WGS 84, gravity anomalies referenced to the WGS 72 Ellipsoidal Gravity Formula, or other in-use ellipsoidal gravity formulas (International 1930, GRS 67, GRS 80, etc.), will need to be referenced to the WGS 84 Ellipsoidal Gravity Formula. The recommended approach is simply to replace the existing gravity anomalies with newly computed WGS 84 gravity anomalies. The latter can be formed using the equation

$$\Delta g_{84} = g + \delta g_A - \gamma_{84} + \text{gravity reduction terms} \qquad (4\text{-}25)$$

where all quantities are defined as in Section 4.4.

This approach, Equation (4-25), has been used to reference the point gravity anomaly files of the DoD Gravity Library to the WGS 84 Ellipsoidal Gravity Formula.

4.5.2 Alternative Approach

When only gravity anomalies are available, the above approach must be replaced by a different conversion process. The general formula for converting gravity anomalies from an in-use ellipsoidal gravity formula to the WGS 84 Ellipsoidal Gravity Formula has the form

$$\Delta g_{84} = \Delta g_{old} + \delta\gamma \qquad (4\text{-}26)$$

where

Δg_{84} = gravity anomaly referenced to the WGS 84 Ellipsoidal Gravity Formula

Δg_{old} = gravity anomaly referenced to an old (in-use) ellipsoidal gravity formula

$\delta\gamma$ = conversion factor (theoretical gravity)

$$\delta\gamma = \gamma_{old} - \gamma_{84} \qquad (4\text{-}27)$$

γ_{old} = theoretical gravity computed using an old (in-use) ellipsoidal gravity formula

γ_{84} = theoretical gravity computed using the WGS 84 Ellipsoidal Gravity Formula.

In Equation (4-26), it is assumed that the measured values of gravity used in forming Δg_{old} have been corrected for atmospheric effects (Section 4.4, above). Also, it is apparent from Equation (4-26) that an existing file of gravity anomalies can be referenced to WGS 84, provided an appropriate expression is available for $\delta\gamma$.

The WGS 72 Ellipsoidal Gravity Formula, when expressed in International System (SI) units, has the form [4.6]:

$$\gamma_{72} = 9.7803327\ (1 + 0.005278994 \sin^2\phi + 0.000023461 \sin^4\phi)\ \mathrm{m\ s^{-2}}. \tag{4-28}$$

This formula was developed using a truncated Chebychev polynomial expansion. Therefore, it is necessary to develop the $\delta\gamma$ conversion from γ_{72} to γ_{84}, using the WGS 84 Ellipsoidal Gravity Formula expressed in the form of a truncated series [4.4]. Operating with this truncated series and Equation (4-28), the following equation was developed for use in Equation (4-26) to convert WGS 72 gravity anomalies to WGS 84 gravity anomalies:

$$\begin{aligned}\delta\gamma = (&0.5929 - 0.0432 \sin^2\phi + 0.1851 \sin^4\phi \\ &- 0.1234 \sin^6\phi - 0.0007 \sin^8\phi) \times 10^{-5}\mathrm{m\ s^{-2}}\ .\end{aligned} \tag{4-29}$$

Additional details are available in [4.4] on the development of Equation (4-29).

Equation (4-29) is listed in Table 4.5 along with analogous expressions for converting gravity anomalies related to the GRS 67, GRS 80, and International 1930 Gravity Formulas to the WGS 84

Ellipsoidal Gravity Formula. Unless the measured gravity values used in forming the original gravity anomalies had the atmospheric correction δg_A applied, this correction must now be applied as part of the conversion-to-WGS 84 process. The relevant equation is:

$$\Delta g_{84} = \Delta g_{old} + \delta\gamma + \delta g_A \ . \tag{4-30}$$

When using Equation (4-30), it's important to recall that δg_A, Equation (4-23), is a function of gravity station (measurement) elevation above mean sea level.

4.6 Comments

As indicated in Section 4.5.1, the point gravity anomaly files of the DoD Gravity Library have been referenced to the WGS 84 Ellipsoidal Gravity Formula and will be officially maintained on that formula. The ellipsoidal gravity formula conversion was accomplished by using the point gravity observations (g) in the DoD Gravity Library Files in the manner prescribed by Equation (4-25). These newly formed WGS 84 point gravity anomalies are being used to generate WGS 84 mean gravity anomalies for the DoD Gravity Library files. Although the files of the DoD Gravity Library will be referenced to the WGS 84 Ellipsoidal Gravity Formula, DMA will maintain the capability to provide requesters with point and mean gravity anomalies referred to any of the Ellipsoidal Gravity Formulas identified in Table 4.5.

REFERENCES

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Table 4.1

WGS 84 Ellipsoidal Gravity Formula

Numerical Form	
$$\gamma = 9.7803267714 \frac{1 + 0.00193185138639 \sin^2\phi}{(1 - 0.00669437999013 \sin^2\phi)^{1/2}} \text{ m s}^{-2}$$	
Analytical Form	
$$\gamma = \gamma_e (1 + k \sin^2\phi)/(1 - e^2 \sin^2\phi)^{1/2}$$	
Definitions	
γ	= Theoretical gravity on (at) the surface of the WGS 84 Ellipsoid
ϕ	= WGS 84 geodetic latitude
γ_e	= Theoretical gravity at the equator ($\phi=0^0$) on the surface of the WGS 84 Ellipsoid
γ_e	= 9.7803267714 m s^{-2}
k	= ($b\gamma_p/a\gamma_e$) - 1
k	= 0.00193185138639
a	= Semimajor axis of the WGS 84 Ellipsoid
a	= 6378137 meters
b	= Semiminor axis of the WGS 84 Ellipsoid
b	= 6356752.3142 meters
γ_p	= Theoretical gravity at the poles ($\phi = 90^0$, $\phi = -90^0$) on the surface of the WGS 84 Ellipsoid
γ_p	= 9.8321863685 m s^{-2}
e^2	= Square of the first eccentricity of the WGS 84 Ellipsoid
e^2	= 0.00669437999013
Gal*	= An acceleration due to gravity of 1 centimeter per second squared
milligal	= 1/1000th of a Gal
milligal	= An acceleration due to gravity of 1 x 10^{-3} centimeters per second squared

* Unit of acceleration named in honor of Galileo Galilei

Table 4.2

The WGS 84 Ellipsoidal Gravity Formula and the Defining and Derived Parameters Used in its Derivation

WGS 84 Ellipsoidal Gravity Formula		
$\gamma = 9.7803267714 \dfrac{1 + 0.00193185138639 \sin^2\phi}{(1 - 0.00669437999013 \sin^2\phi)^{1/2}}$ m s^{-2}		
Parameters	Symbols	Numerical Values
Defining Parameters (WGS 84 Ellipsoid)		
Semimajor axis	a	6378137 m
Normalized second degree zonal gravitational coefficient	$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$
Earth's angular velocity	ω	7292115×10^{-11} rad s^{-1}
Earth's gravitational constant (mass of earth's atmosphere included)	GM	3986005×10^{8} m^3 s^{-2}
Derived Geometric Constants		
Semiminor axis	b	6356752.3142 m
First eccentricity squared	e^2	0.00669437999013
Second eccentricity squared	e'^2	0.00673949674227
$2q_0 = (1 + 3/e'^2)(\arctan e') - 3/e'$	q_0	0.0000733462578707
$q_0' = 3(1 + 1/e'^2).[1 - (1/e') \arctan e'] - 1$	q_0'	0.00268804130046
Derived Physical Constants		
$m = \omega^2 a^2 b/GM$	m	0.00344978600313
Theoretical gravity at the equator	γ_e	9.7803267714 m s^{-2}
Theoretical gravity at the poles	γ_p	9.8321863685 m s^{-2}
$k = (b\gamma_p/a\gamma_e) - 1$	k	0.00193185138639

Table 4.3

Values of Theoretical Gravity
- Surface of WGS 84 Ellipsoid -

Geodetic Latitude (Degrees)	Theoretical Gravity (Milligals)	Geodetic Latitude (Degrees)	Theoretical Gravity (Milligals)	Geodetic Latitude (Degrees)	Theoretical Gravity (Milligals)
0	978032.67714				
1	978034.24974	31	979403.86004	61	981995.59523
2	978038.96567	32	979484.34064	62	982071.68403
3	978046.81924	33	979566.21467	63	982146.01155
4	978057.80102	34	979649.38295	64	982218.48652
5	978071.89781	35	979733.74468	65	982289.01996
6	978089.09264	36	979819.19757	66	982357.52520
7	978109.36485	37	979905.63796	67	982423.91805
8	978132.69006	38	979992.96095	68	982488.11691
9	978159.04021	39	980081.06051	69	982550.04280
10	978188.38360	40	980169.82963	70	982609.61957
11	978220.68492	41	980259.16044	71	982666.77388
12	978255.90532	42	980348.94434	72	982721.43539
13	978294.00240	43	980439.07211	73	982773.53680
14	978334.93030	44	980529.43408	74	982823.01394
15	978378.63975	45	980619.92024	75	982869.80585
16	978425.07811	46	980710.42038	76	982913.85487
17	978474.18944	47	980800.82421	77	982955.10672
18	978525.91458	48	980891.02151	78	982993.51054
19	978580.19118	49	980980.90228	79	983029.01898
20	978636.95383	50	981070.35682	80	983061.58824
21	978696.13407	51	981159.27595	81	983091.17815
22	978757.66052	52	981247.55104	82	983117.75221
23	978821.45895	53	981335.07423	83	983141.27763
24	978887.45237	54	981421.73853	84	983161.72537
25	978955.56108	55	981507.43794	85	983179.07021
26	979025.70285	56	981592.06760	86	983193.29073
27	979097.79292	57	981675.52392	87	983204.36939
28	979171.74416	58	981757.70469	88	983212.29249
29	979247.46717	59	981838.50923	89	983217.05027
30	979324.87035	60	981917.83850	90	983218.63685

Table 4.4

Atmospheric Corrections for Measured Gravity
(When Forming WGS 84 Gravity Anomalies)

Elevation Above Mean Sea Level (km)	Correction δg_A (mgal)	Elevation Above Mean Sea Level (km)	Correction δg_A (mgal)
0	0.87	10	0.23
0.5	0.82	11	0.20
1.0	0.77	12	0.17
1.5	0.73	13	0.14
2.0	0.68	14	0.12
2.5	0.64	15	0.10
3.0	0.60	16	0.09
3.5	0.57	17	0.08
4.0	0.53	18	0.06
4.5	0.50	19	0.05
5.0	0.47	20	0.05
5.5	0.44	22	0.03
6.0	0.41	24	0.02
6.5	0.38	26	0.02
7.0	0.36	28	0.01
7.5	0.33	30	0.01
8.0	0.31	32	0.01
8.5	0.29	$\geq$34	0.00
9.0*	0.27		
9.5	0.25		

* Based on the height of the earth's topography, δg_A values for elevations greater than approximately 9 kilometers (km) above mean sea level are primarily of academic interest. However, some of the higher elevation δg_A values may be of practical value in the future if advances in at-altitude gravity measurement technology continue.

Table 4.5

Equations for Converting Gravity Anomalies to the WGS 84 Ellipsoidal Gravity Formula

From	Conversion Equations $(\delta\gamma)$ *	Maximum Difference
World Geodetic System 1972	$\delta\gamma = (0.5929 - 0.0432 \sin^2\phi + 0.1851 \sin^4\phi - 0.1234 \sin^6\phi - 0.0007 \sin^8\phi) \times 10^{-5}$ m s^{-2}	0.6138 mgal (at $\phi = 68°$)
Geodetic Reference System 1980	$\delta\gamma = (0.0000100 + 0.0000196 \sin^2\phi + 0.0000098 \sin^4\phi - 0.0000196 \sin^6\phi - 0.0000293 \sin^8\phi) \times 10^{-5}$ m s^{-2}	0.000018 mgal ** (at $\phi = 45°$)
Geodetic Reference System 1967	$\delta\gamma = (-0.8271 - 0.1475 \sin^2\phi + 0.1860 \sin^4\phi - 0.1234 \sin^6\phi - 0.0007 \sin^8\phi) \times 10^{-5}$ m s^{-2}	- 0.9127 mgal (at $\phi = 90°$)
International 1930	$\delta\gamma = (16.3229 - 13.8426 \sin^2\phi + 0.3214 \sin^4\phi - 0.1234 \sin^6\phi - 0.0007 \sin^8\phi) \times 10^{-5}$ m s^{-2}	16.3229 mgal (at $\phi = 0°$)

* These conversion equations do not include atmospheric effects. (See Section 4.5.2.)

** Due to the smallness of the difference, this gravity anomaly conversion is unnecessary.

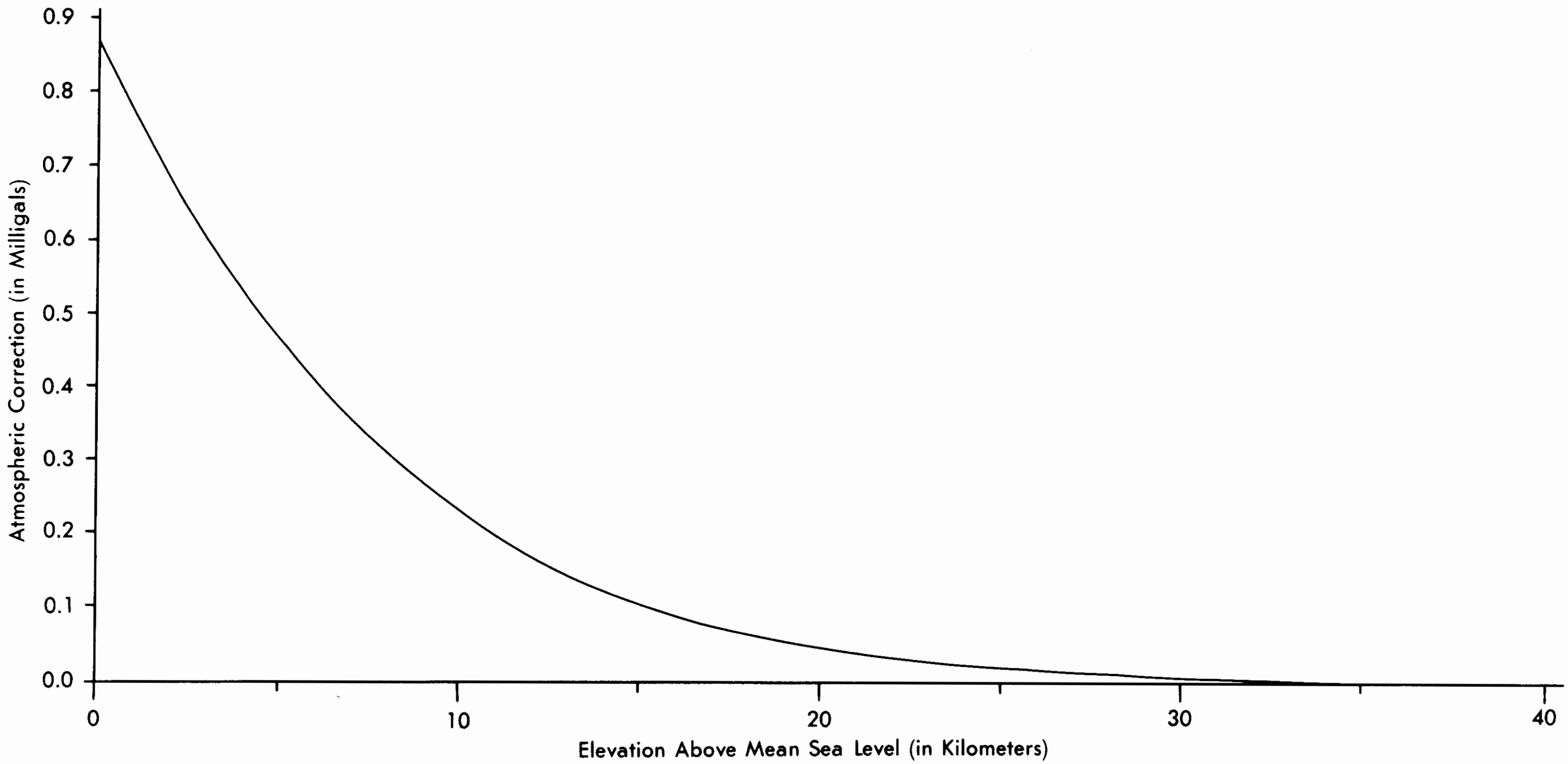

Figure 4.1. Atmospheric Correction for Measured Gravity (When Forming WGS 84 Gravity Anomalies)

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