A Preliminary Gravitational Model to Degree 2160

N.K. Pavlis, S.A. Holmes
Raytheon ITSS Corporation, 1616 McCormick Drive, Upper Marlboro, Maryland 20774, USA
Nikolaos_Pavlis@raytheon.com Fax: +301-883-4140

S.C. Kenyon, D. Schmidt, and R. Trimmer
National Geospatial-Intelligence Agency, Arnold, Missouri 63010, USA

Abstract. The National Geospatial-Intelligence Agency (NGA) of the USA has embarked upon the development of a new Earth Gravitational Model (EGM), to support future realizations of NGA's World Geodetic System. Current plans call for the development of the new EGM (EGM05) by the end of 2005. The new model will be complete to degree and order 2160, and aims at a ±15 cm global Root Mean Square (RMS) geoid undulation error requirement. The new model will combine optimally the gravitational information that is extracted from dedicated geopotential mapping satellite missions (CHAMP, GRACE), with the information contained within a global gravity anomaly database of 5´x5´ resolution. This paper describes the development of a Preliminary Gravitational Model (PGM2004A). We developed PGM2004A by combining the GRACE-only model GGM02S, with a 5´x5´ global gravity anomaly database compiled by NGA. PGM2004A is complete to degree and order 2160, and is accompanied by propagated error maps at 5´x5´ resolution, accounting for the *entire* bandwidth of the model (from degree 2 to degree 2160), for various model-derived gravimetric quantities $(\Delta g, N, \xi, \eta)$. We have evaluated PGM2004A through comparisons with *independent* data including GPS/Leveling data, astronomic deflections of the vertical over the conterminous US (CONUS), and altimeter data from TOPEX. The results of these comparisons indicate that the goal set for EGM05 is well within reach. We summarize in this paper our current status and technical accomplishments, and discuss briefly our next steps towards the development of EGM05.

Keywords. Earth gravitational model, spherical and ellipsoidal harmonics, high-degree expansion.

1 Introduction

The new satellite missions GRACE (JPL, 1998) and GOCE (ESA, 1999) (to be launched in 2006) promise quantum leaps in the accuracy and resolution of *satellite-only* gravitational models. Nevertheless, there is still a need to combine that information with terrestrial gravity and satellite altimetry data in an optimal fashion. Such combination could permit the "seamless" extension of the gravitational spectrum (beyond the resolution recoverable from space techniques), taking advantage of the rich high frequency content in surface gravimetric and altimetric data. Recognizing that need, NGA has embarked upon the development of a new Earth Gravitational Model (EGM05) that will be complete to degree and order 2160. EGM05 is expected to be a *composite* model like EGM96 (Lemoine et al., 1998). A comprehensive combination solution will define the model up to the maximum degree that will be recoverable from GRACE data (~140 or so). Within this comprehensive solution, the GRACE-derived information will be combined with surface gravity data and a Mean Sea Surface, to estimate *simultaneously* gravitational potential coefficients, Dynamic Ocean Topography (DOT) coefficients, and coefficients representing biases within the surface gravity data. Pavlis and Kenyon (2003) discuss some of the design considerations and numerical aspects pertinent to such a combination solution. A complete error covariance matrix will accompany this comprehensive combination solution. Beyond the maximum degree recoverable from GRACE data, and up to degree 2160, EGM05 will be determined from the analysis of a complete, global set of 5´x5´ gravity anomaly data, using block-diagonal and numerical quadrature harmonic analysis techniques.

2 The Development of PGM2004A

While the development of EGM05 is in progress, the currently available data permitted the development of a Preliminary Gravitational Model designated PGM2004A. This model was developed primarily in order to gauge our progress and, more importantly, to identify areas requiring improvement. In terms of data compliment and estimation approach, PGM2004A is very similar to the block-diagonal combination solutions described by Pavlis in Lemoine et al. (1998, Chap. 8). The GRACE-derived satellite-only model GGM02S (Ries, personal communication, 2004) and a 5´x5´ global gravity anomaly database are combined within a block-diagonal least squares adjustment, to yield the coefficients of PGM2004A up to degree and order 2159, as well as their error estimates. This adjustment does not make use of any *a priori* constraints. Spectral weights are used for the surface gravity information, thus affecting a degree-dependent weighting of the 5´ gravity anomaly data, designed to compensate for the neglected error correlations among these data. This approach circumvents many of the shortcomings of the weighting schemes used for corresponding data in previous models. The PGM2004A coefficients of degree 2160 were estimated separately using numerical quadrature harmonic analysis. The main steps of the development of PGM2004A are:

(1) A block-diagonal least squares fit as described in detail by Pavlis in Lemoine et al. (1998, Chap. 8) creates the normal equations implied by the 5´ gravity anomaly data, in terms of ellipsoidal harmonics. This fit also produces a model (designated PGM2004SG) representing *only* the surface gravity information.

(2) The GGM02S coefficients and their standard deviations are converted from spherical to ellipsoidal harmonic representation using Jekeli's (1988) algorithm.

(3) The (strictly diagonal) normal equations of GGM02S (*only* the standard deviations of the model coefficients were available to us) are combined within a least squares adjustment with the surface gravity block-diagonal normal equations. We will use the complete error covariance matrix accompanying the GRACE-derived coefficients, when such a matrix becomes available. This adjustment produces the ellipsoidal harmonic spectra (signal and error) of the combined solution (PGM2004A).

(4) The ellipsoidal harmonic signal and error spectra of PGM2004A are converted to the corresponding spherical ones (Jekeli, 1988).

In the development of PGM2004A, the analytical continuation necessary to reduce the surface free-air Δg to the ellipsoid (a surface of revolution whose symmetries give rise to the block-diagonal normal equation patterns) was omitted. This affects mostly coefficients beyond degree ~90, since lower degree coefficients are dominated by the GRACE information. Analytical continuation will be one of the focal points of our future analysis work.

3 Gravity Anomaly Data Used

A complete global file of 5´x5´ area-mean surface free-air gravity anomalies was compiled by NGA, and became available for this study in May 2004. The gravity anomalies within this "merged" file originate from terrestrial, airborne, and altimetry-derived sources. Within this file, approximately 17% of the Earth's area is occupied by "fill-in" values. These were computed from a composite gravitational model comprised of GGM02S up to degree 100 and EGM96 from degree 101 to 360, plus the gravity anomaly contribution implied by the Residual Terrain Model (RTM) effect (Forsberg, 1984). Figure 1 shows the distribution by source of these 5´ Δg . Table 1 summarizes their essential statistics.

Table 1. Statistics of the 5´ gravity anomaly data (mGal).

Source	% Area	Min.	Max	RMS	RMS σ
ArcGP	4.0	-186.0	235.6	31.3	8.1
Altimetry	67.0	-360.4	377.9	29.2	2.5
Terrestrial	12.3	-223.1	425.3	34.8	6.5
Fill-in	16.7	-318.7	582.5	44.0	18.6
Non Fill-in	83.3	-360.4	425.3	30.2	3.8
All	100.0	-360.4	582.5	32.9	8.4

Notice that the area void of high quality 5´ data (i.e., that occupied by the "fill-in" values) is also the "roughest" area of the gravity anomaly field. This affects the anomaly degree variances recovered from PGM2004A, as we discuss next.

4 The Estimated Spectra

Following the procedure outlined in Section 2, we used the 5´ gravity anomaly data discussed in Section 3, to estimate first (through a block-diagonal least squares fit) the ellipsoidal spectrum representing *only* the surface gravity information (PGM2004SG). Statistics of the residual 5´ gravity anomalies from this fit are shown in Table 2.

Table 2. Statistics of the residual 5´ gravity anomalies produced by PGM2004SG using a block-diagonal least squares fit to degree 2159 and numerical quadrature for degree 2160 (units are mGal).

Source	% Area	Min.	Max	RMS
ArcGP	4.0	-51.1	75.1	1.87
Altimetry	67.0	-117.5	53.3	0.43
Terrestrial	12.3	-57.8	55.1	1.75
Fill-in	16.7	-57.5	95.4	0.93
All	100.0	-117.5	95.4	0.88

These residuals represent primarily signal and/or noise present in the 5´ Δg *beyond* degree and order 2160. Wenzel (1998) reported corresponding RMS residual misfits of ±5.3 mGal, ±5.1 mGal, and ±7.9 mGal for GPM98A, B, and C, respectively (considering in all cases expansions to degree 1799). These values are to be compared with the ±0.88 mGal RMS misfit of our analysis.

The PGM2004SG normal equations were then combined with the GGM02S information to produce the PGM2004A model. Figure 2 shows the gravity anomaly degree variances, for the signal and error spectra involved in the combination solution. Figure 2a shows the low degree portion of these degree variances. Our weighting of the 5´ Δg allows the GGM02S model to dominate the combination solution up to degree and order 90 or so. In this study we considered the error spectrum of GGM02S to be well "calibrated" and did not alter this spectrum in any fashion. The surface gravity information dominates above degree 130. The "transition" from GRACE to surface gravity is effected within a relatively narrow degree range $(90 < n < 130)$, primarily because the error spectra of PGM2004SG and GGM02S intersect at a rather steep angle. Figure 2b shows the spectra for the entire bandwidth of PGM2004A. The signal spectrum of PGM2004A suffers a small "jump" discontinuity at degree 360. This is most likely due to the lack of high quality 5´ data over some of the "roughest" areas of the anomaly field of the Earth, as we discussed in Section 3. PGM2004A's signal spectrum dips below its error spectrum around degree 2010.

In addition to an error spectrum, PGM2004A is accompanied by global 5´x5´ geographic grids of its propagated (commission) error on geoid undulations, gravity anomalies, deflections of the vertical, and gravity disturbances. These grids were computed using the technique of Pavlis and Saleh (this issue), and, *for the first time*, correspond to the entire bandwidth of the model, up to degree and order 2160. Figure 3 shows the geoid undulation propagated error of PGM2004A to degree 2160.

Table 3. Propagated error of various gravimetric quantities implied by PGM2004A to degree 2160.

Region	RMS $\sigma(N)$ (cm)	RMS $\sigma(\Delta g)$ (mGal)	RMS $\sigma(\xi)$ (arcsec)	RMS $\sigma(\eta)$ (arcsec)
"T/P"	15.8	5.3	0.78	0.80
CONUS	15.7	5.4	0.81	0.81
Land	27.1	9.8	1.46	1.46
Ocean	16.3	5.5	0.82	0.83
Globe	20.1	7.0	1.05	1.06

The availability of these geographic grids of the propagated error estimates on various gravimetric functionals, for the *entire* model bandwidth, allows error estimation over any geographic area of interest. These area-specific error estimates can then be compared to the *observed* performance of the model in the particular geographic area, as this performance may be deduced from comparisons with independent data (provided of course that independent test data are available in the particular area). This, in turn, permits optimal weighting and "calibration" of the errors assigned to the gravity data over a particular area to be performed with much higher fidelity than it was possible in previous high-degree gravitational model developments. Table 3 summarizes the propagated errors of various gravimetric quantities implied by PGM2004A to degree 2160, over some geographic areas of interest ("T/P" denotes the ocean area within latitude ±66°). In the next section, we discuss some comparisons with independent data, whose results in certain cases can be compared to the propagated errors shown in Table 3. We emphasize here that the weighting technique used in the development of PGM2004A has not been "calibrated" – it represents our *first* attempt at estimating appropriate weights for the 5´ gravity anomaly data. Our final solution will be considered well "calibrated" when its propagated errors will match its observed performance in comparisons with independent data (after consideration of omission errors and errors associated with the independent data).

5 Model Evaluation

In the following comparisons we consider 4 models:

(1) EGM96 (Lemoine et al., 1998) to $n = 360$.

(2) The composite model "G02S/EGM96" consisting of: GGM02S $(2 \le n \le 100)$ and EGM96 $(101 \le n \le 360)$.

(3) PGM2004A truncated to $n = 360$.

(4) PGM2004A complete to $n = 2160$.

5.1 Comparisons with TOPEX Altimetry

We have used the Sea Surface Heights (SSH) of a 6-year mean track of TOPEX/Poseidon, sampled at the 1 Hz rate. A 1000 m depth "mask" was used in all the comparisons, to exclude shallow water SSH data that may be less reliable (e.g., due to tidal model errors). In addition, we developed a Dynamic Ocean Topography (DOT) model to degree and order 60, based on the GSFC00.1 Mean Sea Surface (Wang, 2001) and the GGM02S-implied geoid.

This DOT model is subtracted from the 1 Hz TOPEX SSH, to yield an estimate of the geoid undulation, that is compared to the corresponding model-derived value. Apart from the residual SSH (499,234 values in total), we also use residual along-track slopes (476,653 values in total), which provide a good measure of the accuracy of model-derived deflections of the vertical over ocean areas. Table 4 summarizes the results of our comparisons.

Table 4. Comparisons with residual SSH and along-track slopes from TOPEX altimeter data.

Model (Nmax)	Residual SSH (cm)		Along-Trk. Slope (″)
	Max ‖•‖	Std. Dev.	Std. Dev.
EGM96 (360)	331	20.7	1.92
G02S/EGM96 (360)	301	19.4	1.92
PGM2004A (360)	297	17.8	1.86
PGM2004A (2160)	177	10.4	0.44

The results of Table 4 show that our combination solution up to degree 360 outperforms both EGM96 and the composite model G02S/EGM96 in this test. The extension of PGM2004A to degree 2160 reduces the standard deviation of the residual SSH by about 60%, and of the residual along-track slopes by about a factor of 4.2. The observed performance of PGM2004A in this test compared to its propagated errors (Table 3, "T/P") indicates that our weighting of the oceanic (mostly altimetric) gravity anomalies may be pessimistic.

5.2 Comparisons with Astronomic Deflections of the Vertical over CONUS

We have used 3561 astronomic deflections of the vertical over the conterminous US (CONUS). Jekeli (1999) describes this dataset, and discusses in detail the appropriate systematic corrections that should be applied to its values before comparing them to model-derived estimates. Table 5 summarizes the results of our comparisons. In this test, we also used the 1´x1´ gravimetric deflections computed at NGS (see www.ngs.noaa.gov/GEOID/DEFLEC99 for details), which we obtained in October 2002.

Table 5. Comparisons with 3561 astronomic deflections of the vertical over CONUS (units are arc seconds).

Model (Nmax)	RMS $\Delta\xi$	RMS $\Delta\eta$
EGM96 (360)	2.80	3.22
G02S/EGM96 (360)	2.79	3.22
PGM2004A (360)	2.75	3.17
PGM2004A (2160)	1.22	1.27
DEFLEC99 (1´→10800)	0.91	0.92

As expected, there is little (but noticeable) distinction in this comparison between the three models extending to degree 360. A major reduction of the RMS differences occurs when PGM2004A is extended to degree and order 2160, and its performance begins to approximate that of the detailed gravimetric deflections computed at NGS. Although the NGS file outperforms PGM2004A by ~30%, we should note here that PGM2004A's performance is somewhat hindered in this test by the omission of the analytical continuation corrections in its development. We will revisit this in future models, which will include the analytical continuation corrections.

5.3 Comparisons with GPS/Leveling Data

Over several years we maintain a global database of GPS/Leveling (GPS/L) data, generously contributed by various colleagues. Currently, our database contains a total of 12684 points, distributed over 46 countries. 6169 of these points are located within CONUS, and 1930 points are in Canada. The majority of our GPS/L data holdings are located over North America, Europe, and Australia, but points over South America, Asia, and Africa are also present in our database. The geographic distribution of these GPS/L stations is uneven, and in many occasions clusters of stations located extremely close to each other are present in the data. This affects significantly the statistics of our comparisons, and may produce misleading results. After careful inspection of the geographic distribution of the GPS/L data within each source, we created a "thinned" version of our database containing 10767 points in total, 4649 of which are in CONUS. Within our database, some of the GPS/L sources provide geoid undulations (N), while others provide height anomalies (ζ). We account for this using Rapp's (1997) formulation. Details of our testing procedure include:

(1) We computed the ζ to N conversion using the same coefficient set as in EGM96, which extends only to degree 360 (Rapp in Lemoine et al., 1998, section 5.2.1). In the future, we will replace this set with one computed from gravitational and elevation models complete to degree 2160.

(2) The GPS/L data over CONUS are segmented by state (see also Smith and Roman, 2001).

(3) We apply a ±2 meter edit to the differences between GPS/L undulations (or height anomalies) and model-derived values.

(4) We compute statistics of GPS/L undulations minus model-derived values after removing a bias, as well as after removing a linear trend.

Table 6. GPS/Leveling Comparisons over CONUS.

Model (Nmax)	Bias Removed		Linear Trend Removed	
	Number Passed Edit	Weighted Std. Dev. (cm)	Number Passed Edit	Weighted Std. Dev. (cm)
EGM96 (360)	4547	21.4	4544	18.1
G02S/EGM96 (360)	4614	19.1	4611	17.7
PGM2004A (360)	4627	18.3	4624	16.9
PGM2004A (2160)	4648	9.7	4645	7.3
G99SSS (1´→10800)	4649	9.1	4646	5.7

Table 7. GPS/Leveling Comparisons Globally.

Model (Nmax)	Bias Removed		Linear Trend Removed	
	Number Passed Edit	Weighted Std. Dev. (cm)	Number Passed Edit	Weighted Std. Dev. (cm)
EGM96 (360)	10571	29.3	10528	25.8
G02S/EGM96 (360)	10645	24.4	10602	22.0
PGM2004A (360)	10657	22.8	10614	20.5
PGM2004A (2160)	10680	15.5	10637	12.6

Tables 6 and 7 summarize our GPS/L comparison results. Table 6 includes the statistics of the differences with the detailed (1´x1´) gravimetric geoid G99SSS (Smith and Roman, 2001). The results of Tables 6 and 7 are quite reassuring. Moving from EGM96 to newer models (expected to be more accurate), the number of points passing the ±2 m editing criterion is monotonically increasing, while the standard deviation of the differences is monotonically decreasing. There is noticeable difference in the performance of the model G02S/EGM96, and that of our combination solution PGM2004A, when both are considered up to degree 360. To degree 2160, PGM2004A performs tantalizing close to the detailed geoid G99SSS. The latter however was developed based on EGM96, and therefore does not benefit the long wavelength improvements brought about by GGM02S. Comparing the *observed* performance of PGM2004A, to the propagated errors of Table 3, we conclude that our weighting of the surface gravity data (at least over the areas where GPS/L data are available) may be pessimistic.

6 Summary and Future Work

This paper described the development and evaluation of a global gravitational model (PGM2004A), complete to degree and order 2160, from the combination of 5´ gravity anomaly data with the satellite-only model GGM02S. PGM2004A is a preliminary solution developed in preparation for EGM05. Its performance indicates that the goals set by NGA for EGM05 are well within reach. Future work will focus on improving the 5´x5´ gravity anomaly database, the error estimation associated with the 5´ data, and their analytical continuation.

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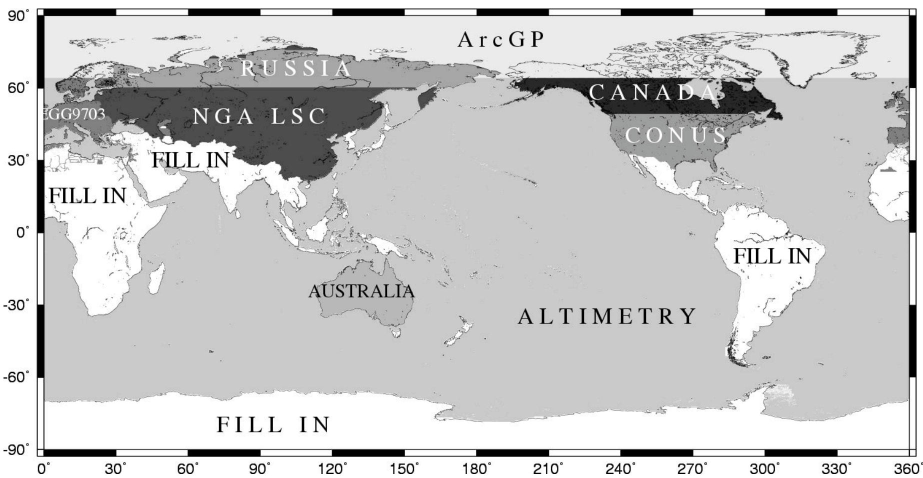

Fig. 1 Geographic Distribution of 5´ Gravity Anomaly Data Sources.

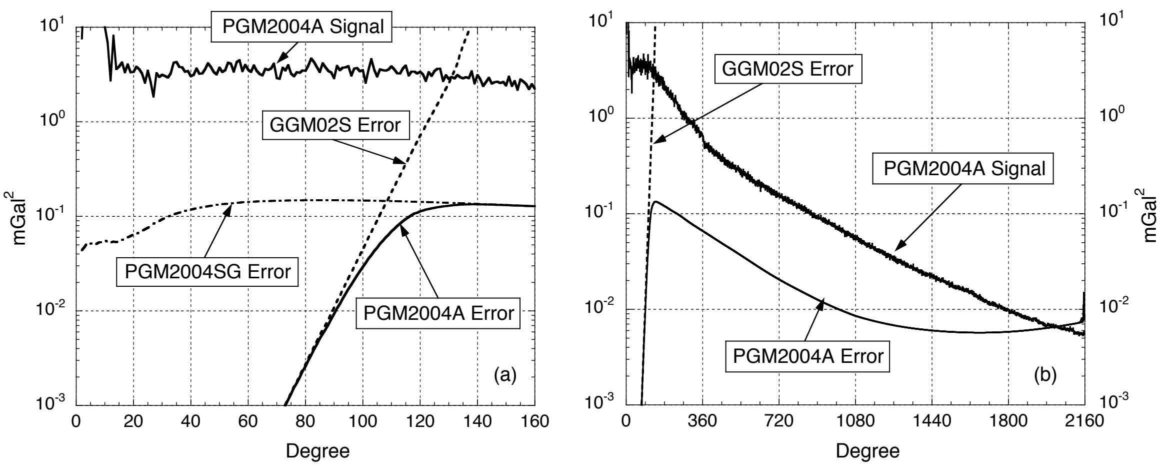

Fig. 2 Gravity Anomaly Degree Variances from Ellipsoidal Harmonic Coefficients: (a) to degree 160, (b) to degree 2160.

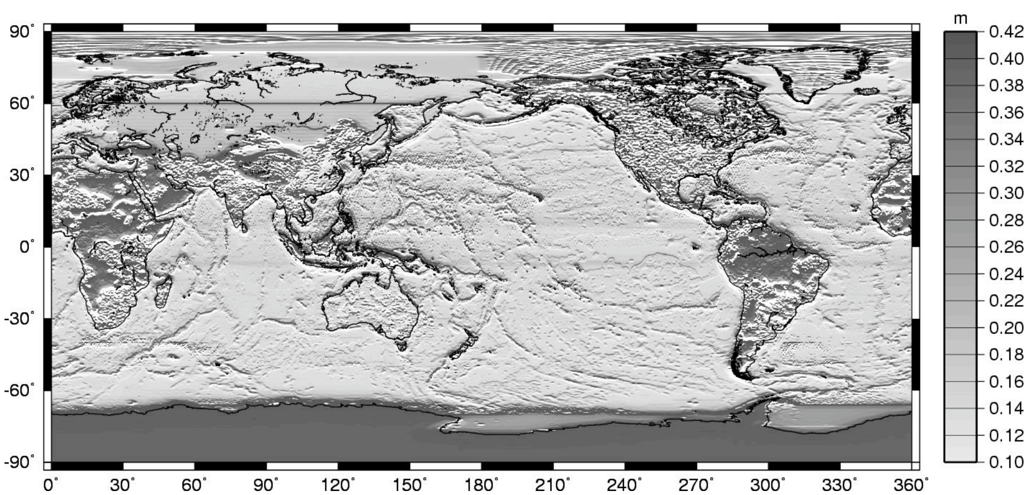

Fig. 3 5´x5´ Geoid Undulation Propagated (Commission) Error from PGM2004A to Degree 2160.