7. METHODS/PROCEDURES FOR OBTAINING WGS 84 COORDINATES

7.1 GENERAL

In mapping, charting, and geodetic (MC&G) activities, it is customary to identify NAD 27 (NAD 83), SAD 69, ED 50, TD, and AGD 66 (AGD 84) as the five major local geodetic systems. However, the number of additional local geodetic systems, or local horizontal datums, in existence in the world today is far more extensive. Counting island and/or astronomic-based datums, the number exceeds several hundred [7.1]. Therefore, it is logical that one of the principal objectives of a world geodetic system is to provide the means whereby local geodetic systems can be referenced to a single earth-centered geodetic system, WGS 84. The basic intent is to have the capability to provide (produce) geodetic coordinates of state-of-the-art accuracy in a single geocentric system suitable for all MC&G applications and DoD operations.

Pursuit of this objective requires the availability (selection) of a geocentric coordinate system and related geocentric coordinates for as many sites, on as many local geodetic systems, as possible. Both geocentric and local geodetic system coordinates are needed at these sites, along with height-above-mean sea level values. Only two satellite (geocentric) coordinate systems, Doppler and laser, were available from which to select the basic reference frame needed for proceeding with the development of WGS 84. Of these two, the Doppler Satellite Coordinate System, NSWC 9Z-2, was the only logical selection since the laser satellite coordinate system, although producing geocentric coordinates of slightly better accuracy, does not have enough sites with both local geodetic and laser-derived coordinates. A total of 1591 Doppler stations (Figure 7.1) satisfied the data acceptability criteria and was available for use in the development of Local Geodetic System-to-WGS 84 Datum Shifts. (By contrast, less than 80 Doppler stations were available for use in the development of Local Geodetic System-to-WGS 72 Datum Shifts.)

The NSWC 9Z-2 Coordinate System and associated network of 1591 Doppler stations were modified in origin, longitude reference, and scale (Table 2.3) to form a WGS 84 station set of improved accuracy and longitude reference. As explained in Chapter 2, the need for these modifications became apparent from the comparison of Doppler-derived quantities with results obtained from satellite laser ranging and Very Long Baseline Interferometry [7.2] [7.3]. As indicated by these comparisons, and as recorded in Table 2.3, the NSWC 9Z-2 Coordinate System could be improved by shifting the equatorial plane of the Doppler Coordinate System downward by 4.5 meters (Figure 2.1), rotating the reference (zero) meridian westward by 0.814 arc second to achieve coincidence with the BIH-defined Zero Meridian (Figure 2.2), and changing Doppler-derived distances (the Doppler scale is too large) by a factor of -0.6×10^{-6} (Figure 2.3). Upon applying these improvements in the manner indicated in Table 2.4, the NSWC 9Z-2 Doppler station set was renamed (became) the WGS 84 station set.

7.2 THE FOUR BASIC METHODS

The four basic methods for obtaining WGS 84 coordinates reflect the geodetic data available, or data situation existing, at the site to be positioned. Depending on the data available, WGS 84 coordinates can be obtained at a site of interest directly in WGS 84 via a satellite point positioning solution, from an NSWC 9Z-2 to WGS 84 coordinate conversion, from a WGS 72-to-WGS 84 coordinate conversion, or by a Local Geodetic System-to-WGS 84 Datum Transformation.

Although it is desirable to have only one set of WGS 84 coordinates for any given site (to prevent error, confusion, and delays), each of the above positioning approaches provides a slightly different set of WGS 84 coordinates. This is especially disconcerting when more than one organization is involved in a project that requires the exchange (or use) of coordinates for the same site or results influenced by them. These four positioning approaches and criteria for their use are discussed in subsequent paragraphs.

7.2.1 Satellite Point Positioning Directly in WGS 84

In preparing for the use of this method, it was necessary to place the Navy Navigation Satellite System (NNSS) on WGS 84. This has been accomplished from a software capability standpoint by NSWC using the approach outlined in Table 7.1.

To obtain the WGS 84 coordinates of a site directly in WGS 84 via satellite point positioning, a ground-based satellite receiver must have "tracked" one or more NNSS satellites and acquired Doppler data at the site (that is being positioned on WGS 84). Local geodetic system coordinates are not required for the site. The Doppler data acquired at the site (preferably 35 or more passes) is used in a satellite point positioning solution to solve for the site's coordinates directly in WGS 84. In such a solution, the WGS 84 NNSS precise ephemerides (based on the WGS 84 EGM through n=m=41) are held fixed (Table 7.2) within estimated error bounds. The Doppler satellite point positioning approach, as just described, is the most accurate and preferred method for obtaining WGS 84 coordinates. It is the recommended approach for positioning new sites or repositioning important sites on WGS 84. Through December 1986, for example, this approach had been used to position the NAVSTAR GPS Master Control and Monitor Stations directly in WGS 84. (It is anticipated that current usage of NNSS Satellites will give way to NAVSTAR GPS Satellites and associated tracking data as the latter system nears full operational capability.)

7.2.2 NSWC 9Z-2 to WGS 84 Transformation

To use this approach, NSWC 9Z-2 coordinates must be known for the site where WGS 84 coordinates are desired, and Doppler tracking data, or appropriate satellite ephemerides, or both, should no longer be available for the site. Otherwise, the site would be positioned directly in WGS 84 using the slightly more accurate satellite point positioning technique. The NSWC 9Z-2 to WGS 84 Transformation listed in Table 2.4 is used to obtain WGS 84 coordinates for a site having such data characteristics. The relevant equations are:

$$\phi_{WGS\ 84} = \phi_{NSWC\ 9Z\text{-}2} + \Delta\phi$$

$$\lambda_{WGS\ 84} = \lambda_{NSWC\ 9Z\text{-}2} + \Delta\lambda \qquad (7\text{-}1)$$

$$H_{WGS\ 84} = H_{NSWC\ 9Z\text{-}2} + \Delta H \ .$$

Table 7.3 shows the difference between WGS 84 coordinates directly determined at the NAVSTAR GPS Master Control and Monitor Stations via Doppler satellite point positioning and WGS 84 coordinates obtained for the same sites using the NSWC 9Z-2 to WGS 84 Transformation. The agreement between the two WGS 84 coordinate sets is reasonably good, with the more accurate coordinates being those determined directly in WGS 84 via satellite point positioning. Since the development of WGS 84 has been completed, and the NNSS has been placed on WGS 84 in relevant software, the NSWC 9Z-2 to WGS 84 Transformation will not be used after WGS 84 is fully implemented.

7.2.3 WGS 72-to-WGS 84 Transformation

Situations arise where only WGS 72 coordinates are available for a site. That is, Doppler satellite tracking data, or appropriate satellite ephemerides, and NSWC 9Z-2 or local geodetic system coordinates are not available for the site (although they may have been available at one time). In such a situation, the preferred approach for obtaining WGS 84 coordinates for the site is to acquire satellite tracking data (at the site) and position it directly in WGS 84 using the satellite point positioning technique. However, it is realistic to presume that use of the preferred positioning approach will not always be practical. In such instances, the WGS 72-to-WGS 84 Transformation listed in Table 7.4 can be used with the following equations to obtain WGS 84 coordinates for the sites:

$$\phi_{WGS\ 84} = \phi_{WGS\ 72} + \Delta\phi$$
$$\lambda_{WGS\ 84} = \lambda_{WGS\ 72} + \Delta\lambda \qquad (7\text{-}2)$$
$$H_{WGS\ 84} = H_{WGS\ 72} + \Delta H \quad .$$

As indicated in Table 7.4, when proceeding directly from WGS 72 coordinates to obtain WGS 84 values, the WGS 84 coordinates will differ from the WGS 72 coordinates due to a shift in the coordinate system origin, a change in the longitude reference, a scale change, and changes in the size and shape of the ellipsoid. These differences are illustrated graphically in Figures 7.2 - 7.5.

The Equations given in Table 7.4 have been used to calculate the difference between WGS 84 and WGS 72 coordinates at five degree intervals of latitude between 90 degrees north and south. The results are listed in Table 7.5. However, the WGS 72-to-WGS 84 Transformation (Table 7.4) providing these results must be used with care since the transformation does not accurately reflect all the changes brought about by WGS 84, carrying forward into the WGS 84 coordinates any shortcomings and uncertainties present in the WGS 72 coordinates. For example, depending on the approach used to obtain the WGS 72 coordinates (Section 9.3), the WGS 84 and WGS 72 geodetic heights at a point will generally differ more than indicated in Table 7.5 due to the difference between the site's WGS 84 and WGS 72 Geoid Heights (Chapter 6). A discussion of the accuracy of the WGS 72-to-WGS 84 Transformation is also provided in Section 9.3. A careful reading of that Section is recommended prior to using Equations (7-2).

7.2.4 Local Geodetic System-to-WGS 84 Datum Transformation Techniques

7.2.4.1 General

Most WGS 84 coordinates needed for MC&G applications and DoD operations will be obtained from a Local Geodetic System-to-WGS 84 Datum Transformation rather than from the three techniques just discussed (Sections 7.2.1 - 7.2.3). The Local Geodetic

System-to-WGS 84 Datum Transformation can be performed either in curvilinear (geodetic) coordinates:

$$\phi_{WGS\ 84} = \phi_{LGS} + \Delta\phi$$
$$\lambda_{WGS\ 84} = \lambda_{LGS} + \Delta\lambda \qquad (7\text{-}3)$$
$$H_{WGS\ 84} = H_{LGS} + \Delta H$$

or, in rectangular coordinates [7.4]:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\ 84} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{LGS} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} \Delta S & \omega & -\psi \\ -\omega & \Delta S & \varepsilon \\ \psi & -\varepsilon & \Delta S \end{bmatrix} \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix}_{LGS} \qquad (7\text{-}4)$$

where ΔS and $(\varepsilon, \psi, \omega)$ represent changes in local geodetic system scale and reference frame orientation, respectively, and (X_0, Y_0, Z_0) are the coordinates of the "initial" (defining) point of the local geodetic system (LGS).

There are several datum transformation formulas for accomplishing the preceding. The most common techniques are, in the curvilinear case, the Standard Molodensky [7.5] or the Abridged Molodensky [7.6], and in the rectangular case, the 3-, 4-, or 7-parameter transformation depending on the availability (and/or reliability) of the transformation parameters. It may be noted that the 3-parameter rectangular case is embedded mathematically in the Molodensky Formulas to save the conversion from geodetic to rectangular coordinates. The Seven Parameter Datum Transformation is discussed briefly in Section 7.2.4.3.1 and analyzed, along with its variants, in Section 7.2.4.4 The Molodensky Datum Transformation Formulas are discussed in Section 7.2.4.3.2.

In addition, the curvilinear and rectangular coordinate datum transformations can be accomplished using Multiple Regression Equations which account for the non-linear distortion in the local geodetic system [7.7]. Multiple Regression Equations (MREs) are discussed in Sections 7.2.4.3.3 and 7.2.4.6.

7.2.4.2 Basic Data

7.2.4.2.1 Local Geodetic System and WGS 84 Coordinates at the Same Sites

To obtain the data needed for performing a Local Geodetic System-to-WGS 84 Datum Transformation, local geodetic system and WGS 84 coordinates are both required at one or more sites within the local geodetic system. The datum shifts (ΔX, ΔY, ΔZ) needed for utilizing the Standard or Abridged Molodensky Datum Transformation Formulas can be obtained if both sets of coordinates are known at one or more stations of the local geodetic system being transformed. Development of Datum Transformation Multiple Regression Equations or the parameters for a Seven Parameter Datum Transformation requires a sufficient number of stations with known coordinates, preferably well distributed, to determine a variable number of regression equation coefficients or seven unknown parameters, respectively.

Doppler stations positioned within the NNSS Coordinate System (NSWC 9Z-2), and with known local geodetic system coordinates, were the basic ingredients in the development of Local Geodetic System-to-WGS 84 Datum Shifts. The WGS 84 coordinates for these Doppler stations were obtained using the NSWC 9Z-2 to WGS 84 Transformation discussed in Section 7.2.2.

The 1591 Doppler stations suitable for use in WGS 84 development were sufficiently dispersed to permit the generation of Local Geodetic System-to-WGS 84 Datum Transformations for 83 different datums. These local geodetic systems (datums) are identified in

Table 7.6 and [7.8]. The semimajor axis (a) and flattening (f) of their associated reference ellipsoids, and other ellipsoids of interest, are also provided in [7.8] along with a set of graphics illustrating the geographical boundary of each of the 83 local geodetic systems. In addition, it should be noted [7.8] that some of the local geodetic systems are used in (extended to) more than one geographical area. For example, ED 50 is used in Europe, parts of Africa and Asia, and for some islands in the Mediterranean Sea. In contrast to the Local Geodetic System-to-WGS 84 Datum Transformation available today for 83 local geodetic systems, Local Geodetic System-to-WGS 72 Datum Transformations were initially available for only 27 local geodetic systems.

The equations for obtaining at Doppler sites the datum shifts needed to compute WGS 84 coordinates at non-Doppler local geodetic network sites from a 3-parameter transformation are

$$X_{WGS\ 84} = X_{LGS} + \Delta X$$
$$Y_{WGS\ 84} = Y_{LGS} + \Delta Y \qquad (7\text{-}5)$$
$$Z_{WGS\ 84} = Z_{LGS} + \Delta Z$$

or

$$\phi_{WGS\ 84} = \phi_{LGS} + \Delta\phi$$
$$\lambda_{WGS\ 84} = \lambda_{LGS} + \Delta\lambda \qquad (7\text{-}6)$$
$$H_{WGS\ 84} = H_{LGS} + \Delta H$$

where ΔX, ΔY, ΔZ and $\Delta\phi$, $\Delta\lambda$, ΔH are termed datum shifts. These individual datum shift values are immediately available at Doppler sites where both WGS 84 and local geodetic system coordinates are known. The next Section discusses how these datum shift values were improved through the use of DMA-developed local geodetic system geoid heights.

7.2.4.2.2 DMA-Developed Local Geodetic System Geoid Heights

As previously stated, the determination of Local Geodetic System-to-WGS 84 Datum Shifts (ΔX, ΔY, ΔZ) is contingent upon having local geodetic system and WGS 84 coordinates available for each Doppler station involved in the datum transformation process. The local geodetic system coordinates (ϕ, λ, H) of a Doppler station are used to compute the site's local geodetic system X,Y,Z values. However, although mean sea level elevations (h) are available for the Doppler sites, local geodetic system geoid heights (N) are usually not available or are unreliable. As a result, the local geodetic heights of Doppler stations determined using such data in the formula $H_{LGS} = N_{LGS} + h$ are of poor quality. This, in turn, leads to poor definitions for:

- The differences (ΔH) between the local and WGS 84 geodetic heights of Doppler stations

- The local geodetic system X, Y, Z coordinates of Doppler stations.

(Also, any degradation inherent in the latter adversely affects the quality of Local Geodetic System-to-WGS 84 Datum Shifts developed in ΔX, ΔY, ΔZ form.) This undesirable situation (due either to the lack of local geoid heights or their poor quality) is widespread, affecting with only a few exceptions all the local geodetic systems for which Local Geodetic System-to-WGS 84 Datum Shifts are desired. For example, although NAD 27 astrogeodetic geoid heights are available for much of North America, they are not of high accuracy due to the poor quality and sparse and uneven distribution of the basic astrogeodetic deflection of the vertical (DOV) components used in their development. The 1978 Version of the ED 50 Geoid [7.9], although of more recent construction than the NAD 27 Astrogeodetic Geoid, is felt to have somewhat the same problems, especially in coastal areas and near the datum boundary.

In developing WGS 84, datum shifts were needed for referencing 83 local geodetic systems to WGS 84. Local geoid heights, regardless of accuracy, were available for only 16 of the 83 datums. Faced with the need for local geoid heights for 83 datums, and the uncertainties known to exist (or strongly suspected) in practically all of the 16 local geoids that were available, DMA decided to develop a local geoid for each local geodetic system for which Local Geodetic System-to-WGS 84 Datum Shifts were needed. This resolved the problem of how to cope with missing and poor quality local geoids and at the same time offered the advantage of having all local geoids based on the same development technique and type of data. These local geoids were prepared by re-referencing the WGS 84 Geoid from the WGS 84 Ellipsoid to the reference ellipsoid and orientation associated with each of the local geodetic systems. The procedure used to form the DMA-developed local geoids is outlined in Table 7.7.

In using DMA-developed local geodetic system geoid heights (N_{DMA}) to obtain Local Geodetic System-to-WGS 84 Datum Shifts of improved accuracy, the Doppler stations' local geodetic system geodetic heights were computed using the formula

$$H_{LGS} = N_{DMA} + h \ . \qquad (7\text{-}7)$$

The local geodetic system geodetic coordinates [with H_{LGS} computed via Equation (7-7)] were then used in the following equations to obtain local geodetic system rectangular coordinates at the Doppler sites [7.10]:

$$\begin{aligned} X_{LGS} &= [R_N + H_{LGS}] \cos \phi \cos \lambda \\ Y_{LGS} &= [R_N + H_{LGS}] \cos \phi \sin \lambda \qquad (7\text{-}8) \\ Z_{LGS} &= [R_N (1-e^2) + H_{LGS}] \sin \phi \ . \end{aligned}$$

In Equation (7-8), R_N and e^2 are defined as in Sections 3.4.7 and 3.4.2, respectively. With the WGS 84 and local geodetic system coordinates of the Doppler sites both now known in X, Y, Z form, Local Geodetic System-to-WGS 84 Datum Shifts (ΔX, ΔY, ΔZ) were formed:

$$X_{WGS\ 84} - X_{LGS} = \Delta X$$

$$Y_{WGS\ 84} - Y_{LGS} = \Delta Y \qquad (7\text{-}9)$$

$$Z_{WGS\ 84} - Z_{LGS} = \Delta Z\ .$$

From the preceding, it is apparent that the DMA-developed local geoid heights are inherently contained in the Local Geodetic System-to-WGS 84 Datum Shifts. This applies whether the Local Geodetic System-to-WGS 84 Transformation takes the form of Molodensky Datum Transformation Formulas which utilize ΔX, ΔY, ΔZ datum shifts (mean or regional values or quantities estimated from ΔX, ΔY, ΔZ contour charts), ΔH coordinate difference contour charts, or ΔH Datum Transformation Multiple Regression Equations. [However, note that there are two exceptions. The $\Delta\phi$ and $\Delta\lambda$ coordinate difference contour charts and the $\Delta\phi$ and $\Delta\lambda$ Datum Transformation Multiple Regression Equations formed using values obtained from Equation (7-6) are unaffected by the use of N_{DMA} values. Longitude is unaffected in any case, as evident from Equation (7-15).] As indicated by Equation (7-7), use of a local geodetic system geodetic height incorporating a DMA-developed local geoid height is needed to obtain an accurate WGS 84 geodetic height for the site in question. The relevant formula is, from Equation (7-6):

$$H_{WGS\ 84} = H_{LGS} + \Delta H$$

However, from a user's perspective, local geodetic system geodetic heights are needed explicitly as input data by only one of the datum transformation techniques that computes $\Delta\phi$, $\Delta\lambda$, ΔH values, the Standard Molodensky Datum Transformation Formulas. And, even then, an approximate local geodetic system geodetic height is suitable for that purpose. That is, a local geoid height need not be available for computing ΔH values, but a local geoid height of good quality, N_{DMA}, is needed in order to obtain H_{LGS} (and thus, $H_{WGS\ 84}$) values of good accuracy.

Local geodetic system geoid heights can be estimated from DMA-prepared contour charts, a geoid height multiple regression equation (Section 7.2.4.6), or interpolated from gridded values. For local geodetic systems of limited geographical extent, such as island datums, the local geoid heights are essentially constant. Therefore, local geoid heights may be listed in tabular form for these small areas. The DMA-developed local geoid height data discussed immediately above is available in [7.8], except for the gridded values and contour charts for many of the local geodetic systems. Gridded local geodetic system geoid heights and/or contour charts, if required, may be requested from DMA. (See PREFACE.)

7.2.4.3 Datum Transformation Formulas

7.2.4.3.1 The Seven Parameter Datum Transformation

The most general approach for converting geodetic data from one geodetic coordinate system to another is a transformation involving three translations, three rotations, a scale change, and accountability for modifications needed due to scale and orientation changes within the geodetic network [7.11]. The seven parameter transformation assumes the geodetic system has a consistent scale and orientation throughout the network. In the seven parameter transformation, the origin of one coordinate system is assumed to be offset from the other, the axes of one coordinate system are assumed to be inclined with respect to the other, and the two systems are assumed to have different scales (i.e., distances between similar points in the two systems have different lengths). The origins and axes of the two systems may be brought into coincidence by three translations and three rotations (Figure 7.6). In addition, a coordinate in the scale of the old system can be determined in the scale of the new system by multiplying the old coordinate by $(1 + \Delta S)$ where ΔS is the difference in scale between the two systems. Therefore, proceeding from Equation (7-4), the equations for obtaining WGS 84 coordinates from a Local Geodetic System-to-WGS 84 seven parameter transformation may be written as

$$
\begin{aligned}
X_{WGS\ 84} &= X_{LGS} + \Delta X_T + \omega(Y-Y_0)_{LGS} - \psi(Z-Z_0)_{LGS} + \Delta S(X-X_0)_{LGS} \\
Y_{WGS\ 84} &= Y_{LGS} + \Delta Y_T - \omega(X-X_0)_{LGS} + \varepsilon(Z-Z_0)_{LGS} + \Delta S(Y-Y_0)_{LGS} \\
Z_{WGS\ 84} &= Z_{LGS} + \Delta Z_T + \psi(X-X_0)_{LGS} - \varepsilon(Y-Y_0)_{LGS} + \Delta S(Z-Z_0)_{LGS}
\end{aligned}
\qquad (7\text{-}10)
$$

or

$$
\begin{aligned}
X_{WGS\ 84} &= X_{LGS} + \Delta X_T + \Delta X_R + \Delta X_S \\
Y_{WGS\ 84} &= Y_{LGS} + \Delta Y_T + \Delta Y_R + \Delta Y_S \\
Z_{WGS\ 84} &= Z_{LGS} + \Delta Z_T + \Delta Z_R + \Delta Z_S
\end{aligned}
\qquad (7\text{-}11)
$$

where the subscripts T, R, and S in Equation (7-11) pertain, respectively, to the changes to these coordinates brought about by performing a coordinate system translation, rotation, and scale change.

In Equation (7-10), the unknowns are ΔX_T, ΔY_T, ΔZ_T, ε, ψ, ω, and ΔS. A minimum of three stations with known WGS 84 and local geodetic system coordinates is required for determining the seven unknown parameters needed to convert the coordinates of a local geodetic system to WGS 84 via this technique. Thirty-five of the 83 local geodetic systems being transformed to WGS 84 have fewer than three Doppler stations co-located at local geodetic system sites. Even when coordinates are available from a sufficient number of co-located sites, a 7-parameter solution may yield less than optimum parameter values due to poor station distribution, distortion within the network, etc. Consequently, rotation and scale parameters are often suppressed (not solved for) in the solution for datum transformation parameters. In such instances, Equation (7-11) may be written as

$$
\begin{aligned}
X_{WGS\ 84} &= X_{LGS} + \Delta X_T \\
Y_{WGS\ 84} &= Y_{LGS} + \Delta Y_T \\
Z_{WGS\ 84} &= Z_{LGS} + \Delta Z_T
\end{aligned}
\qquad (7\text{-}12)
$$

where, as noted earlier when discussing Equation (7-5), ΔX_T, ΔY_T, ΔZ_T are termed datum shifts. The development of these datum shifts (subscript T omitted) was discussed earlier in Section 7.2.4.2.

7.2.4.3.2 The Molodensky Datum Transformation Formulas

The Standard and Abridged Molodensky Datum Transformation Formulas [7.5] [7.6], listed in Table 7.8 along with definitions of the terms that comprise them, are well known. Although the Abridged Molodensky Datum Transformation Formulas have been used more extensively than the Standard Formulas, the latter produce geodetic latitudes at the ellipsoid's surface that are more accurate by 0.6 meter (Table 7.9). Table 7.9 also shows that the Standard Molodensky Datum Tansformation Formulas produce geodetic latitudes and longitudes that are more accurate by approximately one meter at 12 kilometers (approximately 40,000 feet) above the ellipsoid than can be obtained using the Abridged Formulas. Therefore, the Standard Formulas rather than the Abridged Formulas should be used in any new software involving the Molodensky Datum Transformation Formulas.

The ΔX, ΔY, ΔZ datum shifts used to date in the Molodensky Datum Transformation Formulas have normally been mean values. Local Geodetic System-to-WGS 84 Mean Datum Shifts are listed in Table 7.10 for NAD 27 (CONUS), NAD 83 (CONUS), SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84. Figures 7.7-7.15 show the Doppler stations used to determine the mean datum shifts for these local geodetic systems. For informational purposes, the mean datum shifts in Table 7.10 are compared in Table 7.11 with their WGS 72 counterparts. With the exception of Tokyo Datum, and the ΔX values in general, changes in the mean datum shifts are relatively small considering the large increase in the number of Doppler stations used in developing WGS 84 mean datum shifts versus those used for WGS 72. Part of the differences in ΔX and ΔY (Table 7.11) is due to the use of a different longitude reference for WGS 84 than for WGS 72.

Reference [7.8], Section 10, contains Local Geodetic System-to-WGS 84 Mean Datum Shifts and associated data for 83 local geodetic systems. However, due to the triangulation errors that affect most local geodetic systems, use of mean datum shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) in either the Standard or Abridged Molodensky Datum Transformation Formulas produces results that are of insufficient accuracy for many applications. The RMS coordinate differences in Table 7.12, and the histograms of coordinate differences provided for NAD 27 (CONUS) as Figures 7.16a-7.16c, illustrate the errors that occur when mean datum shifts are used for datum transformation purposes. These differences were formed by subtracting WGS 84 coordinates computed using the Abridged Molodensky Datum Transformation Formulas and mean datum shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) from WGS 84 coordinates obtained via the NSWC 9Z-2 to WGS 84 Transformation (Table 2.4). Figures 7.16a-7.16c also appear in [7.8], Section 11, along with similar graphics for NAD 83 (CONUS), SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84.

For many applications, use of either the Standard or Abridged Molodensky Datum Transformation Formulas produces results that are of sufficient accuracy only when localized rather than mean datum shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) are used. Efforts to have localized datum shifts available have led to the preparation of ΔX, ΔY, ΔZ datum shift contour charts. Such contour charts, when used to obtain estimated ΔX, ΔY, ΔZ datum shifts for a specific location, provide a more accurate coordinate conversion, with the Molodensky Datum Transformation Formulas, than can be obtained using mean datum shifts (for the same datum). Contour charts of Local Geodetic System-to-WGS 84 ΔX, ΔY, ΔZ Datum Shifts are provided (Figures 7.17-7.19) for NAD 27 (CONUS). These datum shift contour charts very effectively illustrate the distortion that exists in the NAD 27 geodetic network. However, contour charts of the differences ($\Delta\phi$, $\Delta\lambda$, ΔH) at Doppler stations between local geodetic system and WGS 84 coordinates are easier to use and more visually informative than the ΔX, ΔY, ΔZ datum shift contour charts. Local Geodetic System-to-WGS 84 $\Delta\phi$, $\Delta\lambda$, ΔH contour charts are also provided for NAD 27 (CONUS) (Figures 7.20-7.22). Although the $\Delta\phi$, $\Delta\lambda$, ΔH contour charts provide at a glance the difference between local geodetic system and WGS 84 coordinates at sites of interest, they provide results that are less accurate (due to the contour intervals used in their preparation) than those obtainable through the use of ΔX, ΔY, ΔZ Datum Shift Contour Charts.

Figures 7.17-7.19 and 7.20-7.22 are also provided in [7.8], Sections 13 and 15, respectively, along with similar graphics for NAD 83 (CONUS), SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84. Sections 14 and 16 (of Reference 7.8) contain graphics for the remainder of the 83 local geodetic systems, with a few exceptions.

As stated previously, use of a DMA-developed local geoid height in forming a local geodetic system geodetic height is needed to obtain a WGS 84 geodetic height of good quality. Such a geoid is included in contour chart form (Figure 7.23) for NAD 27 (CONUS). This and similar contour charts are available in Section 17 of [7.8] for NAD 83 (CONUS), SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84. Section 18 contains such graphics for most of the remainder of the 83 local geodetic systems.

The above contour charts, whether in geoid height, datum shift or coordinate difference form, are easily used only in an office environment. The combined need for the greater accuracy provided by localized datum shifts and ease in performing datum transformations in the field has led to the development of geoid height and datum transformation Multiple Regression Equations.

7.2.4.3.3 Datum Transformation Multiple Regression Equations

The development of Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations was initiated for two reasons--the need for better accuracy than could be achieved using the Molodensky Datum Transformation Formulas and mean datum shifts, coupled with the need for a technique more amenable for field use. The multiple regression equations approach essentially automates use of the data used to prepare the WGS 84 minus local geodetic system coordinate difference contour charts ($\Delta\phi$, $\Delta\lambda$, ΔH), making it relatively easy to obtain WGS 84 geodetic coordinates via the relationships

$$\phi_{WGS\ 84} = \phi_{LGS} + \Delta\phi$$
$$\lambda_{WGS\ 84} = \lambda_{LGS} + \Delta\lambda \qquad (7\text{-}13)$$
$$H_{WGS\ 84} = H_{LGS} + \Delta H\ .$$

For $\Delta\phi$, the general form of the Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equation is:

$$\Delta\phi = A_0 + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + \ldots + A_{54}V^9 +$$
$$A_{55}U^9V + A_{56}U^8V^2 + \ldots + A_{64}U^9V^2 + A_{65}U^8V^3 + \ldots + \qquad (7\text{-}14)$$
$$A_{72}U^9V^3 + A_{73}U^8V^4 + \ldots + A_{99}U^9V^9$$

where

$A_0, A_1, \ldots, A_{99}$ = 100 possible coefficients determined in a stepwise multiple regression procedure with U and V each limited to single digit exponents.

$U = k (\phi - \phi_m)$ = normalized geodetic latitude of the computation point

$V = k (\lambda - \lambda_m)$ = normalized geodetic longitude of the computation point

k = scale factor, and degree-to-radian conversion

ϕ, λ = local geodetic latitude and local geodetic longitude (in degrees), respectively, of the computation point

ϕ_m, λ_m = mid-latitude and mid-longitude values, respectively, of the local geodetic datum area (in degrees).

Similar equations are obtained for $\Delta\lambda$ and ΔH by replacing $\Delta\phi$ in the left portion of Equation (7-14) by $\Delta\lambda$ and ΔH, respectively.

Prior to beginning the development process, individual $\Delta\phi$, $\Delta\lambda$, ΔH coordinate differences (WGS 84 minus local geodetic) are formed at each Doppler station within the datum area. The multiple regression procedure [7.7] is then initiated to develop separate equations to fit the $\Delta\phi$, $\Delta\lambda$, and ΔH coordinate differences. The first step of the procedure produces a constant and a variable. The variable will either be a function of ϕ or λ, or both. The procedure then sequentially adds one variable at a time to the equation--the variable that provides the greatest improvement in fitting the coordinate difference. After a variable is added, all variables previously incorporated into the equation are tested and, if one is no longer significant, it is removed. Each addition or removal of a variable is called a "step". This stepwise addition or removal of variables ensures that only significant variables are retained in the final equation. In keeping with Equation (7-14), each variable consists of products of powers of normalized geodetic latitude (U), or normalized geodetic longitude (V), or both; e.g., U^3V^4 is a single variable. The stepwise regression procedure continues until the precision desired for the equation is obtained.

For most local geodetic systems, the Doppler station coverage is sufficient only for developing multiple regression equations that are reliable within the area "covered" by the stations, not from datum boundary-to-datum boundary. However, by introducing auxiliary points into the development process, multiple regression equations can be developed that are reliable throughout the datum area. To obtain the auxiliary points needed in developing Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations, a 1°x1° grid of $\Delta\phi$, $\Delta\lambda$, ΔH coordinate differences was formed by interpolating such values from the five closest Doppler stations. From this 1°x1° grid of values, auxiliary points (each having interpolated $\Delta\phi$, $\Delta\lambda$, ΔH values) were then selected to fill areas of sparse Doppler station coverage and to provide coverage slightly external to the datum boundary. Mean datum shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) and the Standard Molodensky Formulas were used to obtain auxiliary point data ($\Delta\phi$, $\Delta\lambda$, ΔH) for areas having fewer than five Doppler stations. The validity of the resulting datum shift multiple regression equations was then checked by using them to generate $\Delta\phi$, $\Delta\lambda$, ΔH contour charts which were compared visually with similar graphics previously developed from the $\Delta\phi$, $\Delta\lambda$, ΔH values available at the Doppler sites.

When deriving datum transformation multiple regression equations, the basic objective is to fit the known coordinate differences ($\Delta\phi$, $\Delta\lambda$, ΔH) at the Doppler (and auxiliary) stations with equations that require the fewest possible terms to achieve a desired tolerance. The Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations were developed using the criterion that the resulting equations must be capable of reproducing the data from which they were created to an RMS difference $\leq$ 1.5 meters, approximately.

7.2.4.3.4 Datum Transformations Via Direct Coordinate Conversion

Once Local Geodetic System-to-WGS 84 Datum Shifts are formed from Doppler station data and are available, they can be used in a number of ways to obtain WGS 84 coordinates at non-Doppler sites within the datum area. For example, if in ΔX, ΔY, ΔZ form, they can be used either indirectly in the Molodensky Formulas (Section 7.2.4.3.2), and then in Equation (7-6), or inserted directly in Equation (7-5) to obtain WGS 84 coordinates. If in $\Delta\phi$, $\Delta\lambda$, ΔH form, they can be inserted directly in Equation (7-6) to obtain WGS 84 coordinates, or used indirectly to generate Datum Transformation MREs (Section 7.2.4.3.3) which then provide $\Delta\phi$, $\Delta\lambda$, ΔH values for use in Equation (7-6). The quality of the WGS 84 coordinates obtained via Equations (7-5) or (7-6) is improved if Equation (7-7) is also utilized in the manner discussed earlier.

In some situations, WGS 84 X, Y, Z values will have been determined at non-Doppler sites by inserting ΔX, ΔY, ΔZ Local Geodetic System-to-WGS 84 Datum Shifts directly into Equation (7-5), but WGS 84 ϕ, λ, H coordinates are desired. Formulas for obtaining the WGS 84 coordinates of non-Doppler sites in ϕ, λ, H form may be derived from Equation (7-8), ignoring the LGS designation. The resultant formulas are [7.10]:

$$\lambda = \text{Arctan } (Y/X) \qquad (7\text{-}15)$$

$$H = [(X^2 + Y^2)^{1/2}/\cos\phi] - R_N \qquad (7\text{-}16)$$

$$\phi = \text{Arctan } \{[Z/(X^2 + Y^2)^{1/2}] \cdot [1-e^2 R_N/(R_N + H)]^{-1}\}. \qquad (7\text{-}17)$$

Equations (7-16) and (7-17) must be solved iteratively since ϕ and H are both unknowns in these equations. To begin the iteration, it is customary to set H = 0 as a first approximation in Equation (7-17). The value obtained for ϕ is then used in Equation (7-16) to obtain a new value of H to use in Equation (7-17). This iterative process is repeated until the changes exhibited in succeeding values of ϕ and H are acceptably small (e.g., 0.01 arc second and 0.3 meter for ϕ and H, respectively).

This coordinate conversion (datum transformation) approach is somewhat undesirable due to its iterative nature and the uncertainty surrounding the number of iterations needed to achieve an acceptable convergence to values for geodetic latitude and geodetic height. (This was especially true in earlier days due to the lack of computational power.) Depending on the application (earth's surface, near earth, spatial, etc.), the reader may want to review some alternatives to Equations (7-15), (7-16), and (7-17) for possible use in converting X, Y, Z coordinates to ϕ, λ, H coordinates [7.12] - [7.19].

7.2.4.4 Analysis of Local Geodetic System-to-WGS 84 Datum Transformation Techniques

As part of the analysis, seven parameter datum transformation solutions were made for 17 selected local geodetic systems. In selecting these local geodetic systems, care was taken to ensure that small well-adjusted networks such as the Cape Canaveral Datum and large networks affected by distortion, such as NAD 27 (CONUS), were included. Although only seven parameter solutions

($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ε, ψ, ω; ΔS) have been discussed to this point, solutions were also made for six parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ε, ψ, ω), four parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ΔS), and three parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$).

Results from the seven, six, four, and three parameter investigative solutions are given in Table 7.13. These results reveal the variations in the datum transformation parameters that occur when change-of-scale and/or the three rotation parameters are, or are not, included in the least squares solution for the datum transformation parameters. From the table, it is noted that a seven parameter and a six parameter solution:

- Yield rotation angles of the same magnitude; i.e., solving or not solving for a scale change parameter does not affect the solved-for rotation angles.

- Produce similar sets of $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values when ΔS is relatively small. Note, for example, the results for Adindan Datum and NAD 27 (CONUS).

- Produce sets of $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values that may vary considerably when ΔS is large. Note, for example, the results for Old Hawaiian Datum and Ordnance Survey of Great Britain 1936 (OSGB 36).

It is also apparent from Table 7.13 that seven parameter and four parameter solutions generally produce dissimilar sets of $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values since the $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values from the four parameter solutions absorb the effect of not solving for the three rotation parameters. Continuing, it is noted that a four parameter and a three parameter solution:

- Produce similar sets of $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values when ΔS is relatively small. Again, note, for example, results for the Adindan Datum and NAD 27 (CONUS).

- Produce sets of $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values that may vary considerably when ΔS is large. Again, note, for example, the results for Old Hawaiian Datum and OSGB 36.

Also, when the scale change (ΔS) and the three rotation parameters (ε, ψ, ω) are suppressed in the seven parameter least squares solution for the datum transformation parameters, the three translation parameters obtained from the solution are the mean datum shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) often used in the Standard or Abridged Molodensky Datum Transformation Formulas. This is noted by comparing the last entry for a local geodetic system in Table 7.13 with the mean values listed for the same system in Table 7.10.

It is generally assumed that a seven parameter datum transformation is superior to a transformation based on a fewer number of parameters and to transformations based on other techniques. This assumption is investigated by using the seven parameter Local Geodetic System-to-WGS 84 Datum Transformation from Table 7.13 to compute WGS 84 coordinates at sites where "true" WGS 84 coordinates are known. The "true" WGS 84 coordinates at these sites were determined using the NSWC 9Z-2 to WGS 84 Transformation discussed in Section 7.2.2. The RMS differences between the "true" and computed WGS 84 geodetic coordinates are given in Table 7.14. For comparison purposes, Table 7.14 also contains RMS differences between the "true" WGS 84 coordinates and WGS 84 coordinates computed using Local Geodetic System-to-WGS 84 MREs.

From an analysis of Table 7.14, it is noted that:

. A seven parameter datum transformation produces geodetic coordinates that are, in almost all cases:

- Equal to or slightly more accurate than those obtained from a six parameter datum transformation.*

- Equal or superior in accuracy to those obtained from a four parameter datum transformation.*

- Equal or superior in accuracy to those obtained from a three parameter datum transformation.*

. A seven or six parameter datum transformation provides, for example, a much more accurate coordinate conversion than either a four or three parameter datum transformation for Arc Datum 1950. However, this superiority is much less pronounced for many of the local geodetic systems listed in Table 7.14.

. The most important conclusion to be made from Table 7.14 is not whether a seven or six parameter datum transformation is superior to a four or three parameter datum transformation, but involves Datum Transformation Multiple Regression Equations. From Table 7.14, it is apparent that Multiple Regression Equations provide Local Geodetic System-to-WGS 84 Datum Transformations that are equal to or superior in accuracy to the seven, six, four, or three

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* Note: Results from the 7-, 6-, 4-, and 3- parameter datum transformations analyzed here reflect the generation and use of mean datum shifts not localized datum shifts.

parameter datum transformation approaches. By comparing the results for each coordinate, the Multiple Regression Equation technique is seen from the tabular entries to be superior to the seven parameter and three parameter datum transformations for 75 percent and 83 percent of the comparisons, respectively.

7.2.4.5 Selection of Preferred (Recommended) Technique

Many techniques of varying accuracy and ease of use are available (Table 7.15) for transforming local geodetic system coordinates (ϕ, λ, H) to WGS 84 coordinates (ϕ, λ, H). Utilizing the results from Tables 7.13 and 7.14 and other information, the advantages, disadvantages, and main characteristics of these Local Geodetic System-to-WGS 84 Datum Transformation techniques are listed in Table 7.16. Selection of the preferred or recommended technique for converting local geodetic system coordinates to WGS 84 from the large number of possibilities listed in Table 7.15 is possible after some consideration of Table 7.16 and previous developments. The available techniques (options, Table 7.15) are quickly reduced in number by noting those that do not remove the degrading effect of local network distortions on WGS 84 coordinates (Options 7 through 12) and omitting the abridged formulas (Options 2 and 6) in favor of the standard formulas (Options 1 and 5). Options 1 and 4 are omitted since they are visual estimation methods primarily suitable for periodic office use, and Option 5 is eliminated due to the difficulty involved in adapting it for real time or field use. (This difficulty arises with Option 5 because of the need to account in the software for the boundaries associated with each set of regional datum shifts.) Option 13 is eliminated for the same reasons as Option 5 and Options 7 through 12. Thus, the multiple regression equation approach (Option 3) is selected as the best overall technique for accomplishing Local Geodetic System-to-WGS 84 Datum Transformations, subject to the recognition that large errors are possible if the multiple regression equations are used external to their area of derivation (datum boundary) and, recognizing that for certain applications, use of one of the rejected techniques (e.g., Options 1, 4, 5, or 13) may be equally or more appropriate. (Also, see Section 7.4.)

7.2.4.6 Availability of Multiple Regression Equations

Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations are available in [7.8], Sections 19 and 20, for determining WGS 84 geodetic latitudes, longitudes, and heights for all the local geodetic systems that have three or more Doppler stations suitable for datum transformation purposes. In addition, Reference [7.8] contains mean $\Delta\phi$, $\Delta\lambda$, and ΔH values (Section 20, Table 20.46) for those local geodetic systems that have fewer than three Doppler stations suitable for datum transformation purposes. (Mean values are also included in the table for three local geodetic systems that have three Doppler stations.) These tabular mean values retain the $\Delta\phi$, $\Delta\lambda$, ΔH form of the Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations, and may be used in Equation (7-3) to obtain WGS 84 ϕ, λ, H coordinates for local geodetic system non-Doppler sites. Also, as indicated earlier, Local Geodetic System-to-WGS 84 Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) are also available in [7.8], Section 10, for these same local geodetic systems.

As an example, Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations are provided in Table 7.17 for NAD 27 (CONUS). Table 7.17 also lists the precision of the equations. The precision values for the Datum Transformation Multiple Regression Equations were determined by using the equations to compute $\Delta\phi$, $\Delta\lambda$, ΔH values at various locations and then comparing the values in an RMS sense with the known $\Delta\phi$, $\Delta\lambda$, ΔH quantities at the same sites that were used in their derivation. The relevant equation for $\delta\Delta\phi$ is:

$$\delta\Delta\phi_{RMS} = \left\{ \left[\sum_{i=1}^{n} \left(\Delta\phi_i(\text{known}) - \Delta\phi_i(\text{MRE})\right)^2 \right]/n \right\}^{1/2} . \quad (7\text{-}18)$$

Analogous equations pertain to $\delta\Delta\lambda$ and $\delta\Delta H$.

For ease of reference, the precision values for these equations have been grouped in Table 7.18, and Section 19 of [7.8], for NAD 27 (CONUS), SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84. The ED 50 coordinates used in the development of the ED 50-to-WGS 84 Datum Transformation Multiple Regression Equations for the United Kingdom (UK) [7.8] were derived from Ordnance Survey of Great Britain (OSGB) Scientific Network 1980 (SN 80) coordinates [7.20] using a three-parameter shift. This shift was based on the OSGB 36 and ED 50 coordinates of Herstmonceux, the latter derived (for Herstmonceux) through a cross-channel geodetic connection to ED 50 stations in France [7.21]. The ED 50 coordinates used in the development of the ED 50-to-WGS 84 Datum Transformation Multiple Regression Equations for UK/Ireland were also based on OSGB SN 80 adjusted values using a three-parameter shift derived as described above (for the UK) [7.21].

It is important to remember that the entries in Table 7.18 are precision values expressing numerically how well the multiple regression equations produce the basic data on which they are based. This information is more easily discerned, for example, from coordinate differences depicted as in Figures 7.24a - 7.24c for NAD 27 (CONUS). These differences were formed by subtracting WGS 84 coordinates computed using Datum Transformation Multiple Regression Equations from WGS 84 coordinates obtained using the NSWC 9Z-2 to WGS 84 Transformation (Table 2.4). A comparison of Figures 7.16a - 7.16c with Figures 7.24a - 7.24c indicates the better results to be achieved by converting NAD 27 (CONUS) coordinates to WGS 84 using Datum Transformation Multiple Regression Equations instead of Molodensky Datum Transformation Formulas with mean datum shifts. Figures 24.a - 24.c also appear in Section 21 of [7.8] along with similar graphics for SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84. The accuracy with which Local Geodetic System-to-WGS 84 Datum Transformation Multiple Regression Equations produce WGS 84 coordinates is treated in Chapter 9.

Geoid Height Multiple Regression Equations have also been prepared that express the DMA-developed local geodetic system geoid heights. As an example, a Local Geodetic System Geoid Height Multiple Regression Equation is provided in Table 7.19 for NAD 27 (CONUS), along with a precision value. Precision values were determined for the

Local Geodetic System Geoid Height Multiple Regression Equations (MREs) by computing local geodetic system geoid heights at various locations using the MREs and then comparing the values in an RMS sense with local geodetic system geoid heights known at the same sites. The equation used in the comparison has the form:

$$\Delta N_{RMS} = \{ [\sum_{i=1}^{n} (N_i \text{ (known)} - N_i \text{ (MRE)})^2]/n \}^{1/2} \qquad (7\text{-}19)$$

where

N_i(known) = DMA-developed local geodetic system geoid height at Station i

N_i(MRE) = Local geodetic system geoid height computed at Station i using Geoid Height Multiple Regression Equation

n = Number of stations (sites) involved.

In Equation (7-19), ΔN_{RMS} indicates how well the Local Geodetic System Geoid Height MREs reproduce the basic geoid height data used in their development. Precision values computed in this manner have been accumulated in Table 7.20, and in Section 22 of [7.8], for the Local Geodetic System Geoid Height MREs for NAD 27 (CONUS), SAD 69, ED 50 (Western Europe), ED 50 (UK/Ireland), ED 50 (UK Only), TD, AGD 66, and AGD 84.

Multiple Regression Equations are available in Sections 22 and 23 of [7.8] for determining local geodetic system geoid heights for the 83 local geodetic systems for which Local Geodetic System-to-WGS 84 Datum Transformations have been developed, with the following exception. For local geodetic systems of limited geographical extent, such as island datums, the local geoid heights are essentially constant. Therefore, constant geoid heights, suitable for use throughout the areal extent of such small datums, are provided in tabular form (Section 23, Reference 7.8) except as affected by time constraints. These constant geoid heights and Geoid Height Multiple

Regression Equations provide DMA-developed local geodetic system geoid heights appropriate for use in determining WGS 84 coordinates using a Local Geodetic System-to-WGS 84 Datum Transformation.

7.3 SPECIAL LOCAL GEODETIC SYSTEM-TO-WGS 84 DATUM SHIFTS

All Local Geodetic System-to-WGS 84 Datum Shifts previously discussed in this Chapter were developed from the direct use of NSWC 9Z-2 Coordinates available at Doppler stations colocated within each datum area with local geodetic network stations. However, datum shifts are sometimes needed for local geodetic systems which do not have any colocated Doppler and local geodetic network stations. Using various techniques, such non-Doppler derived datum shifts have been developed for the local geodetic systems listed in Table 7.21. Accuracy values cannot be rigorously determined for the Local Geodetic System-to-WGS 84 mean datum shifts recorded in Table 7.21. However, these $\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$ values are the best available to DMA at this time for these local geodetic systems and may be used until replacement values based directly on ground-based satellite tracking data become available. If DMA-developed local geodetic system geoids are required for the datums listed in Table 7.21, such data may be obtained by contacting DMA. (See PREFACE.)

7.4 SUMMARY/CONCLUSIONS/RECOMMENDATIONS

Which of the four basic approaches discussed above for determining WGS 84 coordinates should be used? Adherence to the "data availability" criteria listed in Table 7.22 indicates in general the approach to use. Also, it is assumed that some DoD organizations with only a few important stations to position on WGS 84 will request DMA to directly position these stations in WGS 84 using the Satellite Point Positioning Technique and satellite tracking data acquired at the sites using a ground-based satellite receiver. Although some use will be made of the WGS 72-to-WGS 84 Transformation, most DoD elements will obtain WGS 84 coordinates for their applications via a Local Geodetic System-to-WGS 84 Datum Transformation using either Multiple Regression Equations or the

Standard Molodensky Datum Transformation Formulas. Use of the latter with localized datum shifts estimated from contour charts is the most accurate and perhaps most appropriate approach when the number of positions to be transformed to WGS 84 is small, and if in an office environment. When a large number of positions must be transformed to WGS 84, especially if in near real time with little user interface, and/or the operating environment is worldwide, the former is the best overall technique, while retaining good accuracy. However, unacceptably large errors are possible unless use of the Multiple Regression Equations is rigorously restricted to within the boundary of their derivation (datum) area. For example, as currently developed, they cannot be used to derive WGS 84 positions for the water portions of maps/charts external to the local datum (land) boundary. Alternative techniques are the Standard Molodensky Datum Transformation Formulas, or Option 13 (Table 7.15), used with regional and mean datum shifts. In any case, it is anticipated that in some DoD organizations, for certain applications, the Molodensky Datum Transformation Formulas with mean datum shifts will continue in use temporarily due to computer storage limitations and/or restrictions imposed by existing software, or because such an approach satisfies the MC&G current accuracy requirement. Also, it's important to remember that determination of WGS 84 geodetic heights of good quality via a Local Geodetic System-to-WGS 84 Datum Transformation requires use of the DMA-developed local geodetic system geoid height of the site being positioned on WGS 84. Additional guidance on the selection of a datum transformation technique can be obtained from DMA. (See PREFACE.)

Several datum transformation items of significance appear in this Chapter. Foremost is the fact that, for the first time, a substantial number of local geodetic systems (83) can now be referenced to WGS 84 using Doppler-derived transformations. Also, for the first time, the various local geodetic system geoids of variable accuracy, and those missing entirely, have been replaced in the datum shift generation process by a single DMA-developed local geodetic system geoid of homogeneous structure. In addition, a new multiple regression equation datum transformation technique has been introduced, capable of providing improved accuracy in a field environment when carefully applied. Also, the characteristics of various datum transformation techniques, such as the seven parameter transformation, have been numerically explored.

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Table 7.1

Technique Used to Directly Place the NNSS on WGS 84

1. Compute Precise NNSS Satellite Ephemerides in the NSWC 9Z-2 System (NSWC 10E-1 EGM, GM, and TRANET Station Coordinates)

2. Using Above Ephemerides, Solve for New Noise-Reduced NSWC 9Z-2 Coordinates for TRANET Stations. Analyze and Interpret any Significant Changes.

3. Convert NSWC 9Z-2 TRANET Station Coordinates (From 2, Above) to WGS 84 Using NSWC 9Z-2-To-WGS 84 Conversion

4. Compute Precise NNSS Ephemerides in WGS 84 (Using the Same Set of Doppler Satellite Tracking Data Used in 1, Above, the WGS 84 TRANET Station Coordinates from 3, Above, the WGS 84 GM, and the WGS 84 EGM Through n=m=41)

5. Solve for "Corrected" WGS 84 TRANET Station Coordinates. Analyze and Interpret any Significant Changes.

6. Iterate, if necessary.

NNSS = Navy Navigation Satellite System

TRANET = Tracking Network

Table 7.2

Procedure for Solving Directly for WGS 84 Coordinates*

1. Acquire Doppler Data (35 or More Passes) from Each Site to be Positioned.

2. Compute Precise Ephemerides for "Tracked" Navy Navigation Satellite System (NNSS) Satellites Based on:

   a. Defining Parameters of WGS 84 Ellipsoid

   b. WGS 84 EGM (n=m=41)

   c. WGS 84 Coordinates for Doppler Tracking Network (TRANET) Stations Used in the Generation of the Precise Ephemerides. (These Station Coordinates, Determined When Placing the NNSS on WGS 84, are Held Fixed.)

3. Hold Precise Ephemerides (From 2, Above) Fixed Within Estimated Error Bounds, and Complete Satellite Point Positioning Solution (Using Data From 1, Above) to Directly Obtain Doppler-Derived WGS 84 Coordinates for Site of Interest.

* It is anticipated that current usage of NNSS Satellites will give way to NAVSTAR GPS Satellites and associated tracking data as the latter System nears full operational capability.

Table 7.3

WGS 84 Coordinates Determined Via Doppler Satellite Point Positioning
Minus
WGS 84 Coordinates from NSWC 9Z-2 to WGS 84 Transformation

GPS Monitor Stations	Differences in Geodetic Coordinates		
	Geodetic Latitude	Geodetic Longitude	Geodetic Height
Colorado Springs*	0.6m	-0.9m	0.6m
Hawaii	0.3	0.1	1.4
Kwajalein	-1.1	-0.1	1.9
Ascension	-0.5	-0.5	1.8
Diego Garcia	-1.4	0.5	2.2

*Master Control Station (Falcon Air Force Station, CO)

Table 7.4

Formulas and Parameters
to Transform WGS 72 Coordinates
to WGS 84 Coordinates

Formulas	$\Delta\phi'' = (4.5 \cos \phi) / (a \sin 1'') + (\Delta f \sin 2\phi) / (\sin 1'')$ $\Delta\lambda'' = 0.554$ $\Delta H_m = 4.5 \sin \phi + a\, \Delta f \sin^2 \phi - \Delta a + \Delta r$
Parameters	$\Delta f = 0.3121057 \times 10^{-7}$ $a = 6378135$ m $\Delta a = 2.0$ m $\Delta r = 1.4$ m (See Figure 7.4.)
Instructions	To Obtain WGS 84 Coordinates, Add the $\Delta\phi$, $\Delta\lambda$, ΔH Changes Calculated Using WGS 72 Coordinates to the WGS 72 Coordinates (ϕ, λ, and H, Respectively). Latitude is Positive North and Longitude is Positive East (0° to 360°).

Table 7.5

Difference Between WGS 84
and WGS 72 Geodetic Coordinates*

Degrees	Difference (Meters)		
	Latitude	Longitude	Height
90 N	0.0	0.0	4.1
85	0.4	1.5	4.1
80	0.8	3.0	4.0
75	1.3	4.4	3.9
70	1.7	5.9	3.8
65	2.1	7.2	3.6
60	2.4	8.6	3.4
55	2.8	9.8	3.2
50	3.1	11.0	3.0
45	3.4	12.1	2.7
40	3.6	13.1	2.4
35	3.9	14.0	2.0
30	4.1	14.8	1.7
25	4.2	15.5	1.3
20	4.4	16.1	1.0
15	4.4	16.5	0.6
10	4.5	16.9	0.2
5 N	4.5	17.1	-0.2
0	4.5	17.1	-0.6
5 S	4.4	17.1	-1.0
10	4.4	16.9	-1.4
15	4.2	16.5	-1.8
20	4.1	16.1	-2.1
25	3.9	15.5	-2.5
30	3.7	14.8	-2.8
35	3.5	14.0	-3.1
40	3.3	13.1	-3.4
45	3.0	12.1	-3.7
50	2.7	11.0	-3.9
55	2.4	9.8	-4.2
60	2.1	8.6	-4.3
65	1.7	7.2	-4.5
70	1.4	5.9	-4.7
75	1.1	4.4	-4.8
80	0.7	3.0	-4.8
85	0.4	1.5	-4.9
90 S	0.0	0.0	-4.9

*Applies only when proceeding directly from WGS 72 Coordinates to WGS 84 Coordinates; does not contain the effect of the WGS 84 Earth Gravitational Model and Geoid, nor the effect of Local Geodetic System-to-WGS 84 Datum Shifts being better than Local Geodetic System-to-WGS 72 Datum Shifts.

Table 7.6

Local Geodetic Systems
Related to World Geodetic System 1984

Local Geodetic Systems (Datums)	Associated Reference Ellipsoid
Adindan	Clarke 1880
Afgooye	Krassovsky
Ain el Abd 1970	International
Anna 1 Astro 1965	Australian National
Arc 1950	Clarke 1880
Arc 1960	Clarke 1880
Ascension Island 1958	International
Astro Beacon "E"	International
Astro B4 Sorol Atoll	International
Astro DOS 71/4	International
Astronomic Station 1952	International
Australian Geodetic 1966	Australian National
Australian Geodetic 1984	Australian National
Bellevue (IGN)	International
Bermuda 1957	Clarke 1866
Bogota Observatory	International
Campo Inchauspe	International
Canton Astro 1966	International
Cape	Clarke 1880
Cape Canaveral	Clarke 1866
Carthage	Clarke 1880
Chatham 1971	International
Chua Astro	International
Corrego Alegre	International
Djakarta	Bessel 1841
DOS 1968	International
Easter Island 1967	International
European 1950	International
European 1979	International
Gandajika Base	International
Geodetic Datum 1949	International
Guam 1963	Clarke 1866
GUX 1 Astro	International
Hjorsey 1955	International
Hong Kong 1963	International
Indian	Everest
Ireland 1965	Modified Airy
ISTS 073 Astro 1969	International
Johnston Island 1961	International
Kandawala	Everest
Kerguelen Island	International
Kertau 1948	Modified Everest

Table 7.6 (Cont'd)

Local Geodetic Systems
Related to World Geodetic System 1984

Local Geodetic Systems (Datums)	Associated Reference Ellipsoid
L. C. 5 Astro	Clarke 1866
Liberia 1964	Clarke 1880
Luzon	Clarke 1866
Mahe 1971	Clarke 1880
Marco Astro	International
Massawa	Bessel 1841
Merchich	Clarke 1880
Midway Astro 1961	International
Minna	Clarke 1880
Nahrwan	Clarke 1880
Naparima, BWI	International
North American 1927	Clarke 1866
North American 1983	GRS 80*
Observatorio 1966	International
Old Egyptian	Helmert 1906
Old Hawaiian	Clarke 1866
Oman	Clarke 1880
Ordnance Survey of Great Britain 1936	Airy
Pico de las Nieves	International
Pitcairn Astro 1967	International
Provisional South Chilean 1963**	International
Provisional South American 1956	International
Puerto Rico	Clarke 1866
Qatar National	International
Qornoq	International
Reunion	International
Rome 1940	International
Santo (DOS)	International
São Braz	International
Sapper Hill 1943	International
Schwarzeck	Bessel 1841
South American 1969	South American 1969
South Asia	Modified Fischer 1960
Southeast Base	International
Southwest Base	International
Timbalai 1948	Everest
Tokyo	Bessel 1841
Tristan Astro 1968	International
Viti Levu 1916	Clarke 1880
Wake-Eniwetok 1960	Hough
Zanderij	International

* Geodetic Reference System 1980
** Also known as Hito XVIII 1963

Table 7.7

- Development Procedure -
DMA-Developed Local Geodetic System Geoid Heights

Steps	Description
1	For a Given Geodetic System, Assume All Local Geoid Heights Are Zero at Existing Doppler Stations. Form Local and WGS 84 X, Y, Z Coordinates at Each Doppler Site, and Their Differences (ΔX_o, ΔY_o, ΔZ_o).
2	Compute Local Geodetic System-to-WGS 84 Mean Datum Shifts ($\overline{\Delta X}_o$, $\overline{\Delta Y}_o$, $\overline{\Delta Z}_o$) Using the Formulas $\overline{\Delta X}_o = \sum_n \Delta X_o/n$, $\overline{\Delta Y}_o = \sum_n \Delta Y_o/n$, $\overline{\Delta Z}_o = \sum_n \Delta Z_o/n$ Where n is the Number of Doppler Stations Utilized.
3	Use the Mean Datum Shifts ($\overline{\Delta X}_o$, $\overline{\Delta Y}_o$, $\overline{\Delta Z}_o$) From Step 2 and Appropriate Δa and Δf Values in the Abridged Molodensky Datum Transformation Formula for ΔH (Table 7.8) to Compute $\Delta H \equiv \Delta N$ Values at Each of the Doppler Stations. Δa = WGS 84 Ellipsoid Semimajor Axis Minus That of Local Geodetic System Ellipsoid Δf = WGS 84 Ellipsoid Flattening Minus That of Local Geodetic System Ellipsoid
4	Compute Local Geodetic System Geoid Heights (N_{DMA}) at Each Doppler Station by Applying to the Site's WGS 84 Geoid Height (n=m=180) the Geoid Height Difference (ΔN) Calculated in Step 3: $N_{DMA} = N_{WGS\ 84} - \Delta N$. (Note That the Sense of This Equation is WGS 84 Minus Local Where N_{DMA} is Local.)

Table 7.8

Standard and Abridged Molodensky Datum Transformation Formulas
- Local Geodetic System to WGS 84 -

1. The Standard Molodensky Formulas

$$\Delta\phi'' = \{- \Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi$$
$$+ \Delta a \, (R_N e^2 \sin\phi \cos\phi)/a + \Delta f \, [R_M(a/b) + R_N(b/a)] \sin\phi \cos\phi\}$$
$$\cdot \, [(R_M + H) \sin 1'']^{-1}$$

$$\Delta\lambda'' = [- \Delta X \sin\lambda + \Delta Y \cos\lambda] \cdot [(R_N+H) \cos\phi \sin 1'']^{-1}$$

$$\Delta H_m = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi$$
$$- \Delta a \, (a/R_N) + \Delta f \, (b/a) \, R_N \sin^2\phi$$

2. The Abridged Molodensky Formulas

$$\Delta\phi'' = [- \Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + (a\Delta f + f\Delta a) \sin 2\phi]$$
$$\cdot \, [R_M \sin 1'']^{-1}$$

$$\Delta\lambda'' = [- \Delta X \sin\lambda + \Delta Y \cos\lambda] \cdot [R_N \cos\phi \sin 1'']^{-1}$$

$$\Delta H_m = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi + (a\Delta f + f\Delta a) \sin^2\phi - \Delta a$$

3. Definition of Terms in the Molodensky Formulas

ϕ, λ, H = geodetic coordinates (old ellipsoid)

ϕ = geodetic latitude. The angle between the plane of the geodetic equator and the ellipsoidal normal at a point (measured positive north from the geodetic equator, negative south).

λ = geodetic longitude. The angle between the plane of the Zero Meridian and the plane of the geodetic meridian of the point (measured in the plane of the geodetic equator, positive east from the Zero Meridian through 360°).

H = geodetic height (ellipsoidal height). The distance of a point from the ellipsoid measured from the surface of the ellipsoid along the ellipsoidal normal to the point.*

H = N + h

*This parameter does not appear in the Abridged Molodensky Formulas.

Table 7.8 (Cont'd)

Standard and Abridged Molodensky Datum Transformation Formulas
- Local Geodetic System to WGS 84 -

N = geoid/ellipsoid separation. The distance of the geoid above (+N) or below (-N) the ellipsoid. (Use of a DMA-developed local geoid height in forming a local geodetic system geodetic height is needed to obtain a WGS 84 geodetic height of good quality.)

*h = distance of a point from the geoid (elevation of the point above or below mean sea level); positive above mean sea level, negative below mean sea level.

$\Delta\phi$, $\Delta\lambda$, ΔH = corrections to transform local geodetic system coordinates to WGS 84 ϕ, λ, H values. The units of $\Delta\phi$ and $\Delta\lambda$ are arc seconds ("); the units of ΔH are meters (m).

ΔX, ΔY, ΔZ = shifts between centers of the local geodetic system and WGS 84 Ellipsoids; corrections to transform local geodetic system-related rectangular coordinates (X, Y, Z) to WGS 84-related X, Y, Z values.

a = semimajor axis of the local geodetic system ellipsoid.

*b = semiminor axis of the local geodetic system ellipsoid.

*b/a = 1 - f

f = flattening of the local geodetic system ellipsoid.

Δa, Δf = differences between the semimajor axis and flattening of the local geodetic system ellipsoid and the WGS 84 Ellipsoid, respectively.

e = first eccentricity.

$e^2 = 2f - f^2$

R_N = radius of curvature in the prime vertical.

$R_N = a/(1-e^2 \sin^2\phi)^{1/2}$

R_M = radius of curvature in the meridian.

$R_M = a(1-e^2)/(1-e^2 \sin^2 \phi)^{3/2}$

NOTE: All Δ-quantities are formed by subtracting local geodetic system ellipsoid values from WGS 84 Ellipsoid values.

* This parameter does not appear in the Abridged Molodensky Formulas.

Table 7.9

Differences Between Use Of Standard Versus Abridged Molodensky Datum Transformation Formulas To Compute Local Geodetic System-To-WGS 84 Datum Shifts

Geodetic Coordinates	Maximum Coordinate Difference* Height of Point Above Ellipsoid (H**)										
	H = 0	H = 2	H = 4	H = 6	H = 8	H = 10	H = 12	H = 14	H = 16	H = 18	H = 28
Latitude	0.59 m	0.59 m	0.59 m	0.62 m	0.71 m	0.86 m	1.02 m	1.17 m	1.33 m	1.45 m	2.22 m
Longitude	0.00	0.15	0.31	0.46	0.62	0.77	0.93	1.05	1.20	1.36	2.13
Height	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20

* From Analysis Involving 21 Local Geodetic Systems

** H in Kilometers (e.g., H = 12 Kilometers is Approximately 40,000 Feet)

Table 7.10

Local Geodetic System-To-WGS 84

Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) for Major Datums

Local Geodetic System	Country (or Geographic Area)	Mean Datum Shifts			Number of Doppler Stations Used
		$\overline{\Delta X}$ (m)	$\overline{\Delta Y}$ (m)	$\overline{\Delta Z}$ (m)	
NAD 27*	CONUS	- 8	160	176	405
NAD 83*	CONUS	0	0	0	216
SAD 69	South America	- 57	1	- 41	84
ED 50**	Western Europe	- 87	- 98	-121	85
ED 50***	UK/Ireland	- 86	- 96	-120	47
ED 50***	UK	- 86	- 96	-120	40
TD	Japan/Korea/Okinawa	-128	481	664	13
AGD 66	Australia	-133	- 48	148	105
AGD 84	Australia	-134	- 48	149	90

* CONUS = Contiguous United States

** Western Europe = Austria, Belgium, Denmark, Finland, France, FRG (Federal Republic of Germany), Gibraltar, Greece, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland

*** These ED 50 coordinates developed from OSGB Scientific Network (SN 80) coordinates

Table 7.11

Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) and Their Differences
- Local Geodetic System-to-WGS 84 and WGS 72 -

Local Geodetic Systems	Mean Datum Shifts 1984			Mean Datum Shifts 1972			Differences (Absolute)			Doppler Stations Used	
	$\overline{\Delta X}$ (m)	$\overline{\Delta Y}$ (m)	$\overline{\Delta Z}$ (m)	$\overline{\Delta X}$ (m)	$\overline{\Delta Y}$ (m)	$\overline{\Delta Z}$ (m)	$\overline{\delta\Delta X}$ (m)	$\overline{\delta\Delta Y}$ (m)	$\overline{\delta\Delta Z}$ (m)	1984	1972
NAD 27*	- 8	160	176	- 22	157	176	14	3	0	405	38
NAD 83*	0	0	0	---	---	---	--	--	-	216	--
SAD 69	- 57	1	- 41	- 77	3	-45	20	2	4	84	1
ED 50**	- 87	- 98	-121	- 84	-103	-127	3	5	6	85	8
ED 50***	- 86	- 96	-120	---	---	---	--	--	-	47	--
ED 50****	- 86	- 96	-120	---	---	---	--	--	-	40	--
TD	-128	481	664	-140	516	673	12	35	9	13	4
AGD 66	-133	- 48	148	-122	- 41	146	11	7	2	105	4
AGD 84	-134	- 48	149	---	---	---	--	--	-	90	--

* CONUS = Contiguous United States

** Western Europe = Austria, Belgium, Denmark, Finland, France, FRG (Federal Republic of Germany), Gibraltar, Greece, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland

*** United Kingdom (UK)/Ireland; these ED 50 coordinates developed from OSGB Scientific Network 1980 (SN 80) coordinates

**** UK only; these ED 50 coordinates developed from OSGB Scientific Network 1980 (SN 80) coordinates

Table 7.12

RMS Differences
Between "Observed (Doppler)" WGS 84 Coordinates and Computed* WGS 84 Coordinates

Local Geodetic System	Country or Geographic Area	RMS Differences			Number of Doppler Stations Used
		Latitude	Longitude	Height	
NAD 27	CONUS	±4.3 m	±2.7 m	±2.0 m	405
NAD 83	CONUS	0.7	0.9	0.7	216
SAD 69	South America	5.5	10.0	3.7	84
ED 50	Western Europe**	2.3	4.9	1.8	85
ED 50***	UK/Ireland	0.7	0.9	1.4	47
ED 50***	UK	0.8	1.0	1.6	40
TD	Japan/Korea/Okinawa	4.6	5.0	2.0	13
AGD 66	Australia	2.1	1.9	1.6	105
AGD 84	Australia	1.1	1.1	1.6	90

CONUS = Contiguous United States; UK = United Kingdom

* Computed Using Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) in the Abridged Molodensky Datum Transformation Formulas

** Western Europe = Austria, Belgium, Denmark, Finland, France, FRG (Federal Republic of Germany), Gibraltar, Greece, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland

*** These ED 50 coordinates developed from OSGB Scientific Network 1980 (SN 80) coordinates

Table 7.13

- Investigative Results -
Local Geodetic System-to-WGS 84
Datum Transformations

Local Geodetic System	Number of Parameters Solved for in Solution	Local Geodetic System-to-WGS 84 Datum Transformation Parameters						
		ΔX (m)	ΔY (m)	ΔZ (m)	ε	ψ	ω	ΔS*
Adindan	7	-163	- 18	229	0.390"	-0.653"	-0.163"	-0.1985
	6	-164	- 19	228	0.390	-0.653	-0.163	- --
	4	-161	- 12	207	- --	- --	- --	-0.1985
	3	-162	- 12	206	- --	- --	- --	- --
Arc 1950	7	-144	- 73	-305	4.931	2.989	-1.475	-1.7098
	6	-153	- 78	-302	4.931	2.989	-1.475	- --
	4	-134	- 85	-298	- --	- --	- --	-1.7098
	3	-143	- 90	-294	- --	- --	- --	- --
Australian Geodetic 1966	7	-127	- 50	153	0.058	-0.018	-0.089	1.2065
	6	-131	- 45	149	0.058	-0.018	-0.089	- --
	4	-128	- 52	152	- --	- --	- --	1.2065
	3	-133	- 48	148	- --	- --	- --	- --
Cape Canaveral	7	- 1	147	182	-0.025	0.324	-0.156	0.5109
	6	- 1	150	180	-0.025	0.324	-0.156	- --
	4	- 2	147	183	- --	- --	- --	0.5109
	3	- 2	150	181	- --	- --	- --	- --
Ireland 1965	7	496	-132	575	1.102	0.339	0.803	5.2412
	6	516	-134	602	1.102	0.339	0.803	- --
	4	486	-119	584	- --	- --	- --	5.2412
	3	506	-122	611	- --	- --	- --	- --

* Multiply all entries in this column by 10^{-6}

Table 7.13 (Cont'd)

- Investigative Results -
Local Geodetic System-to-WGS 84
Datum Transformations

Local Geodetic System	Number of Parameters Solved for in Solution	Local Geodetic System-to-WGS 84 Datum Transformation Parameters						
		ΔX (m)	ΔY (m)	ΔZ (m)	ε	ψ	ω	ΔS*
European 1950 (Western Europe)	7	-102	-102	-129	0.413"	-0.184"	0.385"	2.4664
	6	- 93	-100	-117	0.413	-0.184	0.385	---
	4	- 97	- 99	-113	---	---	---	2.4664
	3	- 87	- 98	-121	---	---	---	---
Geodetic Datum 1949	7	55	- 17	184	-0.773	0.122	-0.745	5.9218
	6	83	- 20	209	-0.773	0.122	-0.745	---
	4	56	- 18	184	---	---	---	5.9218
	3	84	- 22	209	---	---	---	---
Indian (Thailand and Vietnam)	7	227	803	274	-0.444	-0.645	-0.353	6.5931
	6	218	842	235	-0.444	-0.645	-0.353	---
	4	223	797	291	---	---	---	6.5931
	3	214	836	303	---	---	---	---
North American 1927 (CONUS)	7	- 4	166	183	-0.257	0.341	-0.088	0.3723
	6	- 4	165	184	-0.257	0.341	-0.088	---
	4	- 8	161	175	---	---	---	0.3723
	3	- 8	160	176	---	---	---	---
North American 1983 (CONUS)	7	0.42	0.95	-0.62	-0.012	-0.006	0.012	-0.1364
	6	0.29	0.29	-0.10	-0.012	-0.006	0.012	---
	4	0.24	0.79	-0.86	---	---	---	-0.1364
	3	0.12	0.12	-0.35	---	---	---	---

* Multiply all entries in this column by 10^{-6}

Table 7.13 (Cont'd)

- Investigative Results -
Local Geodetic System-to-WGS 84
Datum Transformations

Local Geodetic System	Number of Parameters Solved for in Solution	Local Geodetic System-to-WGS 84 Datum Transformation Parameters						
		ΔX (m)	ΔY (m)	ΔZ (m)	ε	ψ	ω	ΔS*
Old Hawaiian	7	200	-292	7	0.394"	7.859"	0.764"	7.9200
	6	157	-310	25	0.394	7.859	0.764	---
	4	105	-268	-199	---	---	---	7.9200
	3	61	-285	-181	---	---	---	---
Ordnance Survey of Great Britain 1936	7	446	- 99	544	-0.945	-0.261	-0.435	-20.8927
	6	368	- 95	436	-0.945	-0.261	-0.435	---
	4	453	-114	538	---	---	---	-20.8927
	3	375	-111	431	---	---	---	---
Provisional South American 1956	7	-319	209	-343	-1.372	0.561	-0.522	7.0436
	6	-303	169	-343	-1.372	0.561	-0.522	---
	4	-304	215	-375	---	---	---	7.0436
	3	-288	175	-375	---	---	---	---
Puerto Rico	7	61	77	92	-6.939	1.263	0.255	-11.8252
	6	30	142	68	-6.939	1.263	0.255	---
	4	42	7	- 77	---	---	---	-11.8252
	3	11	72	-101	---	---	---	---
South American 1969	7	- 56	- 3	- 38	0.123	-0.569	-0.158	-0.6412
	6	- 57	0	- 37	0.123	-0.569	-0.158	---
	4	- 56	- 2	- 42	---	---	---	-0.6412
	3	- 57	1	- 41	---	---	---	---

* Multiply all entries in this column by 10^{-6}

Table 7.13 (Cont'd)

- Investigative Results -
Local Geodetic System-to-WGS 84
Datum Transformations

Local Geodetic System	Number of Parameters Solved for in Solution	Local Geodetic System-to-WGS 84 Datum Transformation Parameters						
		ΔX (m)	ΔY (m)	ΔZ (m)	ε	ψ	ω	ΔS*
Tokyo	7	-128	499	672	-0.139"	0.124"	-0.316"	-2.4798
	6	-120	489	663	-0.139	0.124	-0.316	---
	4	-136	491	672	---	---	---	-2.4798
	3	-128	481	664	---	---	---	---
Wake-Eniwetok 1960	7	41	459	- 36	-7.757	1.064	-11.381	-22.6566
	6	181	429	- 58	-7.757	1.064	-11.381	---
	4	- 38	83	- 17	---	---	---	-22.6566
	3	101	52	- 39	---	---	---	---

* Multiply all entries in this column by 10^{-6}

Table 7.14

Difference Between "True" and Computed WGS 84 Coordinates
as a Function of Datum Transformation Parameters (or Technique) Used

Local Geodetic System	- RMS Differences - "True" Minus Computed Coordinates			Datum Transformation Parameters or Technique Used	Number of Points Used in Computations
	$\Delta\phi$	$\Delta\lambda$	ΔH		
Adindan	± 2.3 m	± 3.4 m	± 1.8 m	7	25
	2.3	3.4	1.8	6	..
	3.4	3.6	2.4	4	..
	3.5	3.7	2.4	3	..
	4.7	2.8	2.1	MREs	25
Arc 1950	8.9	11.7	3.8	7	41
	9.1	11.7	3.8	6	..
	13.0	25.5	4.0	4	..
	13.1	25.5	3.9	3	..
	2.7	2.2	1.7	MREs	41
Australian Geodetic 1966	1.7	1.6	1.4	7	105
	2.1	1.9	1.4	6	..
	1.7	1.6	1.4	4	..
	2.1	1.9	1.4	3	..
	1.2	1.3	1.5	MREs	105
Cape Canaveral	0.8	0.8	1.2	7	16
	0.8	0.8	1.2	6	..
	0.8	0.8	1.2	4	..
	0.9	0.8	1.2	3	..
	0.8	0.9	1.1	MREs	16

MREs = Multiple Regression Equations

Table 7.14 (Cont'd)

Difference Between "True" and Computed WGS 84 Coordinates
as a Function of Datum Transformation Parameters (or Technique) Used

Local Geodetic System	- RMS Differences - "True" Minus Computed Coordinates			Datum Transformation Parameters or Technique Used	Number of Points Used in Computations
	Δφ	Δλ	ΔH		
Ireland 1965	± 0.5 m	± 0.5 m	± 0.4 m	7	7
	0.6	0.7	0.4	6	..
	0.6	0.9	0.5	4	..
	0.6	1.1	0.5	3	..
	0.7	1.1	0.6	MREs	7
European 1950 (Western Europe)	2.1	3.1	1.7	7	85
	2.7	3.7	1.7	6	..
	2.5	4.0	1.8	4	..
	2.4	4.9	1.8	3	..
	1.4	1.9	1.6	MREs	85
Geodetic Datum 1949	1.2	1.5	1.8	7	14
	2.8	1.8	1.8	6	..
	1.6	2.6	1.8	4	..
	3.7	1.8	1.8	3	..
	0.9	0.8	1.6	MREs	14
Indian (Thailand and Vietnam)	5.7	5.2	1.3	7	14
	5.6	5.7	1.3	6	..
	5.5	5.5	1.4	4	..
	5.5	6.0	1.4	3	..
	4.1	1.9	2.4	MREs	14

MREs = Multiple Regression Equations

Table 7.14 (Cont'd)

Difference Between "True" and Computed WGS 84 Coordinates
as a Function of Datum Transformation Parameters (or Technique) Used

Local Geodetic System	- RMS Differences - "True" Minus Computed Coordinates			Datum Transformation Parameters or Technique Used	Number of Points Used in Computations
	$\Delta\phi$	$\Delta\lambda$	ΔH		
North American 1927 (CONUS)	± 3.6 m	± 2.7 m	± 2.0 m	7	405
	3.7	2.7	2.0	6	..
	4.3	2.8	1.9	4	..
	4.3	2.7	1.9	3	..
	1.6	1.6	1.7	MREs	405
North American 1983 (CONUS)	0.6	0.9	0.6	7	216
	0.6	0.9	0.6	6	..
	0.6	0.9	0.6	4	..
	0.6	0.9	0.6	3	..
Old Hawaiian	4.4	2.6	1.6	7	13
	4.3	3.1	1.6	6	..
	4.8	3.4	2.8	4	..
	4.4	4.2	2.7	3	..
	1.6	0.8	1.1	MREs	13
Ordnance Survey of Great Britain 1936	1.7	1.7	1.2	7	38
	7.7	3.0	1.3	6	..
	2.1	2.0	1.3	4	..
	7.9	3.0	1.4	3	..
	0.6	0.9	1.0	MREs	38
Provisional South American 1956	16.1	12.9	9.1	7	65
	21.5	9.0	10.1	6	..
	16.9	15.9	8.7	4	..
	23.1	12.0	8.3	3	..
	6.2	3.3	7.4	MREs	65

MREs = Multiple Regression Equations

Table 7.14 (Cont'd)

Difference Between "True" and Computed WGS 84 Coordinates
as a Function of Datum Transformation Parameters (or Technique) Used

Local Geodetic System	- RMS Differences - "True" Minus Computed Coordinates			Datum Transformation Parameters or Technique Used	Number of Points Used in Computations
	$\Delta\phi$	$\Delta\lambda$	ΔH		
Puerto Rico	± 1.0 m	± 1.4 m	± 1.4 m	7	11
	1.0	1.8	1.4	6	..
	2.0	1.5	1.9	4	..
	1.9	1.9	1.9	3	..
	1.2	1.2	1.3	MREs	11
South American 1969	6.9	6.3	3.6	7	84
	6.8	6.5	3.6	6	..
	5.7	9.8	3.6	4	..
	5.5	10.0	3.6	3	..
	2.6	2.6	3.3	MREs	84
Tokyo	4.8	4.5	1.5	7	13
	4.6	5.0	1.5	6	..
	4.8	4.5	1.9	4	..
	4.6	5.0	1.9	3	..
	5.3	3.6	2.7	MREs	13
Wake-Eniwetok 1960	0.5	1.2	1.3	7	7
	0.9	1.2	1.3	6	..
	0.7	1.4	1.6	4	..
	0.7	1.6	1.6	3	..
	0.6	0.6	0.7	MREs	7

MREs = Multiple Regression Equations

Table 7.15

Techniques Available for Transforming
Local Geodetic System Coordinates (ϕ, λ, H) to WGS 84 Coordinates (ϕ, λ, H)

Options	Techniques
1	Standard Molodensky Datum Transformation Formulas and Localized ΔX, ΔY, ΔZ Datum Shifts (Interpolated from ΔX, ΔY, ΔZ Contour Charts).
2	Abridged Molodensky Datum Transformation Formulas and Localized ΔX, ΔY, ΔZ Datum Shifts (Interpolated from ΔX, ΔY, ΔZ Contour Charts).
3*	Datum Transformation Multiple Regression Equations.*
4	Interpolate $\Delta\phi$, $\Delta\lambda$, ΔH Values from $\Delta\phi$, $\Delta\lambda$, ΔH Contour Charts.
5	Standard Molodensky Datum Transformation Formulas with Regional ΔX, ΔY, ΔZ Datum Shifts.
6	Abridged Molodensky Datum Transformation Formulas with Regional ΔX, ΔY, ΔZ Datum Shifts.
7	Seven Parameter Datum Transformation Formulas ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ε, ψ, ω ; ΔS)
8	Six Parameter Datum Transformation Formulas ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ε, ψ, ω)
9	Four Parameter Datum Transformation Formulas ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ΔS)
10	Three Parameter Datum Transformation Formulas ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$)
11	Standard Molodensky Datum Transformation Formulas with Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) .
12	Abridged Molodensky Datum Transformation Formulas with Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$).
13	Coordinate Conversion (X, Y, Z to ϕ, λ, H ; See Section 7.2.4.3.4.)

* This Technique for Calculating $\Delta\phi$, $\Delta\lambda$, ΔH Requires no Graphics or Interpolation Activities While Still Providing Good Accuracy.

Table 7.16

Advantages, Disadvantages, and/or Characteristics
of
Available Local Geodetic System-to-WGS 84 Datum Transformation Techniques

Datum Transformation Techniques	Advantages (and/or Characteristics)	Disadvantages (and/or Characteristics)
Standard Molodensky	. The most accurate of all the datum transformation techniques when localized datum shifts (ΔX, ΔY, ΔZ) are used. . Simple formulas; technique utilizes localized or mean datum shifts (translation parameters) and ellipsoid parameter differences (Δa, Δf). (When mean datum shifts are used, this technique is analogous to the three parameter datum transformation below.)	. Unless localized datum shifts (ΔX, ΔY, ΔZ) are used, this technique does not properly account for variable scale, survey errors, and distortions within the local geodetic network. However, some of the effect is reflected (absorbed) in the mean datum shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$). . Does not give the accuracy desired throughout the network when mean datum shifts are used unless the network is homogeneous (well-defined, well-adjusted).
Abridged Molodensky	. Formulas more simple than the Standard Molodensky; technique utilizes localized or mean datum shifts (translation parameters) and ellipsoid parameter differences (Δa, Δf).	. Slightly less accurate than the Standard Molodensky Formulas, and has the same disadvantages. (Due to the computational and storage capabilities available today, its use is no longer recommended.)
Multiple Regression Equations	. Technique that best accounts for variable scale, survey errors, and distortions within the local geodetic network, with the exception of the Standard and Abridged Molodensky Formulas when localized datum shifts are used. . Can be used throughout the local geodetic network and to the datum boundary if care has been used in determining the Multiple Regression Equation coefficients.	. Requires more terms than other datum transformation techniques if the local geodetic network is strongly nonhomogeneous. . If inadvertently used external to area of derivation (representation), large errors are possible.

Table 7.16 (Cont'd)

Advantages, Disadvantages, and/or Characteristics
of
Available Local Geodetic System-to-WGS 84 Datum Transformation Techniques

Datum Transformation Techniques	Advantages (and/or Characteristics)	Disadvantages (and/or Characteristics)
Coordinate Conversion (X, Y, Z to ϕ, λ, H)	. Simple formulas; technique utilizes localized or mean datum shifts (translation parameters) and local geodetic system ellipsoid parameters (a, e^2).	. Has the same disadvantages as the Standard Molodensky Formulas; also has the disadvantage of being an iterative process with the attendant uncertainty regarding the number of iterations required to achieve convergence.
Seven Parameters	. Simple Formulas . Theoretically, a very correct technique since the formulas utilize least squares developed translation parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$), rotation parameters (ε, ψ, ω), and a scale change parameter (ΔS).	. Does not achieve in practice its theoretical capability since it does not account for variable scale, survey errors, and distortions within the local geodetic network. However, the technique does determine and reflect mean scale and rotation parameters.
Six Parameters	. Simple Formulas . The formulas utilize least squares developed translation and rotation parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$; ε, ψ, ω).	. Slightly less accurate than the seven parameter datum transformation.
Four Parameters	. Simple Formulas; [The formulas utilize least squares developed translation parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) and a scale change parameter (ΔS)].	. Less accurate than the seven and six parameter datum transformations. . Does not give the accuracy desired throughout the network unless the network is homogeneous (well-defined, well-adjusted).

Table 7.16 (Cont'd)

Advantages, Disadvantages, and/or Characteristics
of
Available Local Geodetic System-to-WGS 84 Datum Transformation Techniques

Datum Transformation Techniques	Advantages (and/or Characteristics)	Disadvantages (and/or Characteristics)
Three Parameters	. Simple Formulas; [The formulas utilize least squares developed translation parameters ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$)].	. Less accurate than the seven and six parameter datum transformations. . Slightly less accurate than the four parameter datum transformation. . Does not give the accuracy desired throughout the network unless the network is homogeneous (well-defined, well-adjusted).

Table 7.17

Local Geodetic System-to-WGS 84
Datum Transformation Multiple Regression Equations ($\Delta\phi$, $\Delta\lambda$, ΔH)
- North American Datum 1927 (NAD 27)* to WGS 84 -

$$
\begin{aligned}
\Delta\phi'' = \; & 0.16984 - 0.76173\ U + 0.09585\ V + 1.09919\ U^2 - 4.57801\ U^3 - 1.13239\ U^2V + 0.49831\ V^3 \\
& - 0.98399\ U^3V + 0.12415\ UV^3 + 0.11450\ V^4 + 27.05396\ U^5 + 2.03449\ U^4V + 0.73357\ U^2V^3 \\
& - 0.37548\ V^5 - 0.14197\ V^6 - 59.96555\ U^7 + 0.07439\ V^7 - 4.76082\ U^8 + 0.03385\ V^8 + 49.04320\ U^9 \\
& - 1.30575\ U^6V^3 - 0.07653\ U^3V^9 + 0.08646\ U^4V^9
\end{aligned}
$$

$$
\begin{aligned}
\Delta\lambda'' = \; & - 0.88437 + 2.05061\ V + 0.26361\ U^2 - 0.76804\ UV + 0.13374\ V^2 - 1.31974\ U^3 - 0.52162\ U^2V \\
& - 1.05853\ UV^2 - 0.49211\ U^2V^2 + 2.17204\ UV^3 - 0.06004\ V^4 + 0.30139\ U^4V + 1.88585\ UV^4 \\
& - 0.81162\ UV^5 - 0.05183\ V^6 - 0.96723\ UV^6 - 0.12948\ U^3V^5 + 3.41827\ U^9 - 0.44507\ U^8V \\
& + 0.18882\ UV^8 - 0.01444\ V^9 + 0.04794\ UV^9 - 0.59013\ U^9V^3
\end{aligned}
$$

$$
\begin{aligned}
\Delta H_m = \; & - 36.526 + 3.900\ U - 4.723\ V - 21.553\ U^2 + 7.294\ UV + 8.886\ V^2 - 8.440\ U^2V - 2.930\ UV^2 \\
& + 56.937\ U^4 - 58.756\ U^3V - 4.061\ V^4 + 4.447\ U^4V + 4.903\ U^2V^3 - 55.873\ U^6 + 212.005\ U^5V \\
& + 3.081\ V^6 - 254.511\ U^7V - 0.756\ V^8 + 30.654\ U^8V - 0.122\ UV^9
\end{aligned}
$$

* Contiguous United States (CONUS)

Table 7.17 (Cont'd)

Local Geodetic System-to-WGS 84
Datum Transformation Multiple Regression Equations ($\Delta\phi$, $\Delta\lambda$, ΔH)
- North American Datum 1927 (NAD 27)* to WGS 84 -

In the preceding equations:

$U = K (\phi - 37)$

$V = K (\lambda - 265)$

$K = 0.05235988$

ϕ = geodetic latitude, local geodetic system, in degrees and decimal part of a degree; positive north (0° to 90°), negative south (0° to -90°)

λ = geodetic longitude, local geodetic system, in degrees and decimal part of a degree; positive east from 0° to 360°

The preceding equations reproduced Doppler-derived WGS 84 geodetic coordinates at 447, 425, and 428 comparison points to the following root-mean-square (RMS) differences, respectively:

ϕ: ±1.3 m; λ: ±1.3 m; H (geodetic height): ±1.2 m

Test Case:

Input data for NAD 27 test point:	ϕ = 34°47'08.833"N	$\Delta\phi$ = 0.356"
	λ = 273°25'07.825"E	$\Delta\lambda$ = 0.080"
		ΔH = -38.06 m

*Contiguous United States (CONUS)

Table 7.18

RMS Differences Between "True" WGS 84 Coordinates
and WGS 84 Coordinates Computed Using Local Geodetic System-to-WGS 84
Datum Transformation Multiple Regression Equations

Local Geodetic System	- RMS Differences - "True" Minus Computed Coordinates			Comparison Points (Average Number)
	$\Delta\phi$	$\Delta\lambda$	ΔH	
NAD 27*	±1.3 m	±1.3 m	±1.2 m	433
SAD 69	1.4	1.5	1.8	133
ED 50 (Western Europe)	1.1	1.0	1.4	106
ED 50 (UK/Ireland)**	0.7	1.0	1.0	68
ED 50 (UK Only)**	0.8	1.0	1.0	57
TD	2.1	2.3	2.8	56
AGD 66	1.2	1.3	1.1	132
AGD 84	1.0	1.0	1.5	112

* Contiguous United States (CONUS)

** European Datum 1950 coordinates developed from OSGB Scientific Network 1980 (SN 80) coordinates

Table 7.19

Multiple Regression Equation for DMA-Developed
Local Geodetic System Geoid Heights
- North American Datum 1927 (NAD 27)* -

$$
\begin{aligned}
N_m = {} & 5.068 - 11.570\ U - 8.574\ V + 27.839\ U^2 - 51.911\ UV + 29.496\ V^2 + 28.343\ U^3 + 24.481\ UV^2 \\
& + 11.424\ V^3 + 132.550\ U^3V - 110.232\ U^2V^2 + 41.018\ UV^3 - 64.953\ V^4 - 128.293\ U^3V^2 \\
& + 51.241\ UV^4 - 4.326\ V^5 + 104.097\ U^4V^2 - 128.031\ U^3V^3 + 110.694\ U^2V^4 + 36.330\ V^6 \\
& + 243.149\ U^6V - 15.790\ U^2V^5 - 38.043\ UV^6 - 40.277\ U^2V^6 + 2.746\ UV^7 - 7.321\ V^8 - 394.404\ U^9 \\
& - 927.540\ U^8V + 63.390\ U^4V^5 + 10.626\ UV^8 - 0.520\ UV^9 - 117.207\ U^8V^4 + 16.352\ U^5V^8
\end{aligned}
$$

<u>In the preceding equation:</u>

$U = K\ (\ \phi - 37\ )$
$V = K\ (\ \lambda - 265\ )$
$K = 0.05235988$
ϕ = geodetic latitude, local geodetic system, in degrees and decimal part of a degree; positive north (0° to 90°), negative south (0° to -90°)
λ = geodetic longitude, local geodetic system, in degrees and decimal part of a degree; positive east from 0° to 360°
N = DMA-developed NAD 27 geoid height (in meters) referenced to the Clarke 1866 Ellipsoid

The preceding equation reproduced the geoid heights from which it was generated to a root-mean-square (RMS) difference of ±1.3 meters; 444 sites involved in the comparison.

<u>Test Case:</u>

Input data for NAD 27 test point:	ϕ = 34°47'08.833"N	N = 8.63 m (Clarke 1866 Ellipsoid)
	λ = 273°25'07.825"E	

* Contiguous United States (CONUS); Clarke 1866 Ellipsoid

Table 7.20

RMS Difference With Which Geoid Height Multiple Regression Equations
Reproduce DMA-Developed Local Geodetic System Geoid Heights

Local Geodetic System	- RMS Difference - Original Minus Computed (ΔN)	Number of Comparison Points
NAD 27*	± 1.3 m	444
SAD 69	2.0	140
ED 50 (Western Europe)	1.0	116
ED 50 (UK/Ireland)**	0.4	66
ED 50 (UK Only)**	0.5	55
TD	1.5	51
AGD 66	1.4	134
AGD 84	1.2	109

* Contiguous United States (CONUS)

** European Datum 1950 coordinates developed from Ordnance Survey of Great Britain (OSGB) Scientific Network 1980 (SN 80) coordinates

Table 7.21

Non-Doppler Derived Datum Transformation Parameters
- Local Geodetic Systems to WGS 84 -

Local Geodetic Systems	Reference Ellipsoids and Parameter Differences*			Transformation Parameters*		
	Name	Δa (m)	$\Delta f \times 10^4$	ΔX(m)	ΔY(m)	ΔZ(m)
BUKIT RIMPAH Bangka and Belitung Islands (Indonesia)	Bessel 1841	739.845	0.10037483	-384	664	-48
CAMP AREA ASTRO Camp McMurdo Area, Antarctica	International	-251	-0.14192702	-104	-129	239
G. SEGARA Kalimantan Island (Indonesia)	Bessel 1841	739.845	0.10037483	-403	684	41
HERAT NORTH Afghanistan	International	-251	-0.14192702	-333	-222	114
HU-TZU-SHAN Taiwan	International	-251	-0.14192702	-634	-549	-201
TANANARIVE OBSERVATORY 1925 Madagascar	International	-251	-0.14192702	-189	-242	-91
YACARE Uruguay	International	-251	-0.14192702	-155	171	37

* WGS 84 minus local geodetic system

Table 7.22

Methods For Obtaining WGS 84 Coordinates
For
Sites of Interest

Data Situation at Sites to be Positioned on WGS 84	WGS 84 Positioning Method
1. Local geodetic system coordinates are available for the site, and perhaps WGS 72 coordinates. (No Doppler data and appropriate satellite ephemerides, or NSWC 9Z-2 coordinates, are available for the site.)	Use Local Geodetic System-to-WGS 84 Datum Transformation Formulas, ignoring any WGS 72 coordinates available.* (Use of a local geodetic system geodetic height that incorporates a DMA-developed local geoid height is needed to obtain a WGS 84 geodetic height of good quality.)
2. Only WGS 72 coordinates are available for the site. (No Doppler data and appropriate satellite ephemerides, or NSWC 9Z-2, or local geodetic system coordinates are available for the site.)	Re-positioning of the site via the next approach (below) is preferred since the WGS 72-to-WGS 84 Transformation does not reflect the effect of the WGS 84 Earth Gravitational Model and Geoid. However, Case 1 of the footnote is applicable here.
3. Ground-based satellite receivers have "tracked" NNSS satellites and acquired Doppler data at the site. (Local geodetic system coordinates may or may not be available for the site, and are not required.)	Use satellite point positioning and the Doppler data acquired at the site to solve for the site's coordinates directly in WGS 84. [The NNSS (satellite precise ephemerides, TRANET station coordinates, etc.) has been placed on WGS 84, in relevant software.] (See Table 7.2, this report, and Section 24 of Reference 7.8.)

NOTE: It is anticipated that situations will arise where both geocentric and relative positioning of many local sites to high accuracy on WGS 84 will be required. A weapon system test range is a good example of such a situation. When such situations arise, the preceding positioning procedures should not automatically be applied. Rather, the situation should first be reviewed to determine if a local area geodetic adjustment of all the sites on WGS 84 would be a more appropriate procedure. Defense Mapping Agency personnel are available for assisting in the performance of test range-related geodetic analyses and determining if any new surveys are required.

* However, if the WGS 72 coordinates were determined from a satellite point positioning solution (Case 1), or from a Local Geodetic System-to-WGS 72 Datum Transformation utilizing localized datum shifts (Case 2), use of the WGS 72-to-WGS 84 Transformation (Table 7.4) will provide WGS 84 coordinates of good quality. (See Section 26 and Table 27.1 of Reference 7.8 and Sections 7.2.3 and 9.3. of this report.)

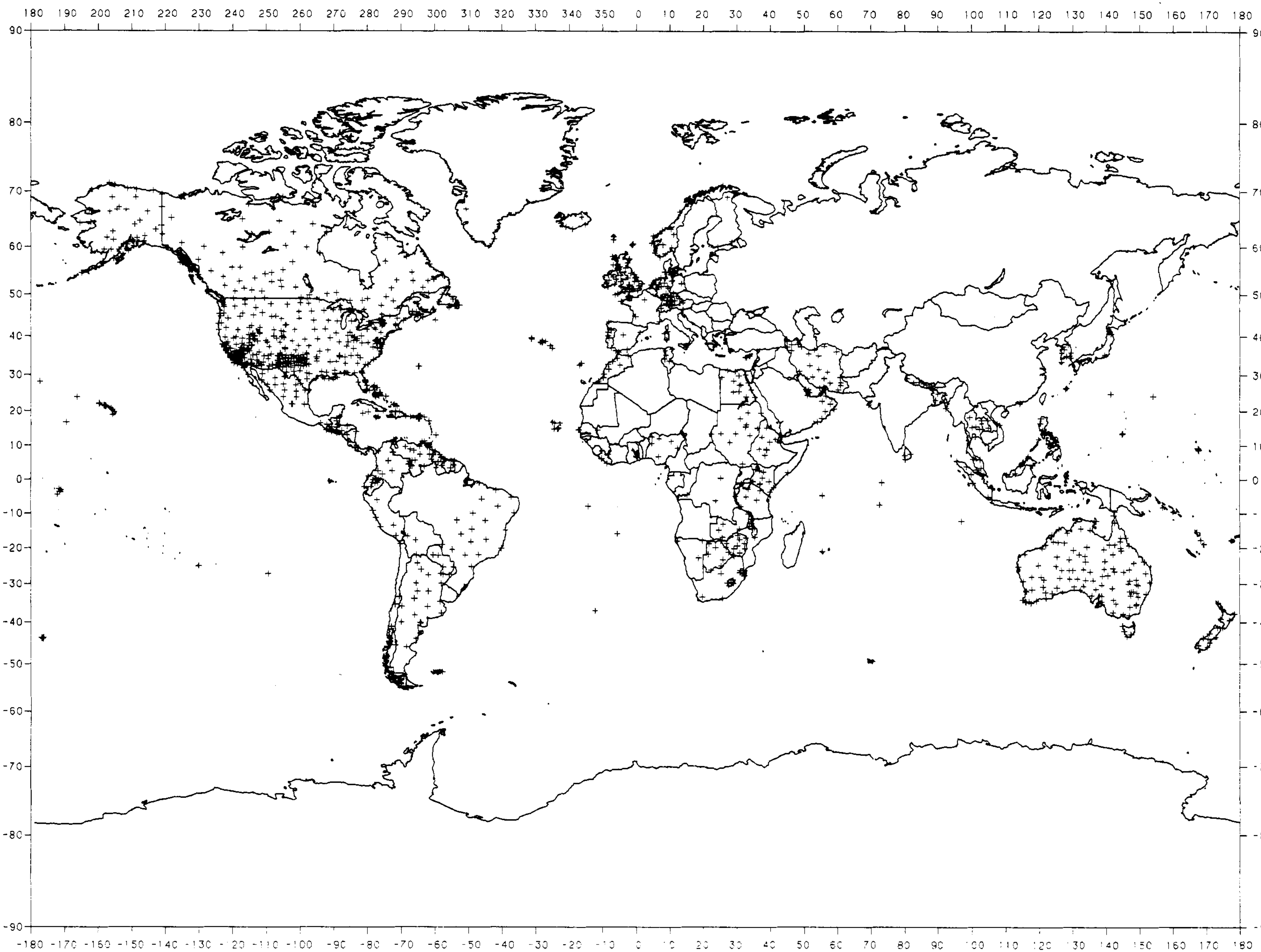


Figure 7.1. Doppler Stations Used in WGS 84 Development

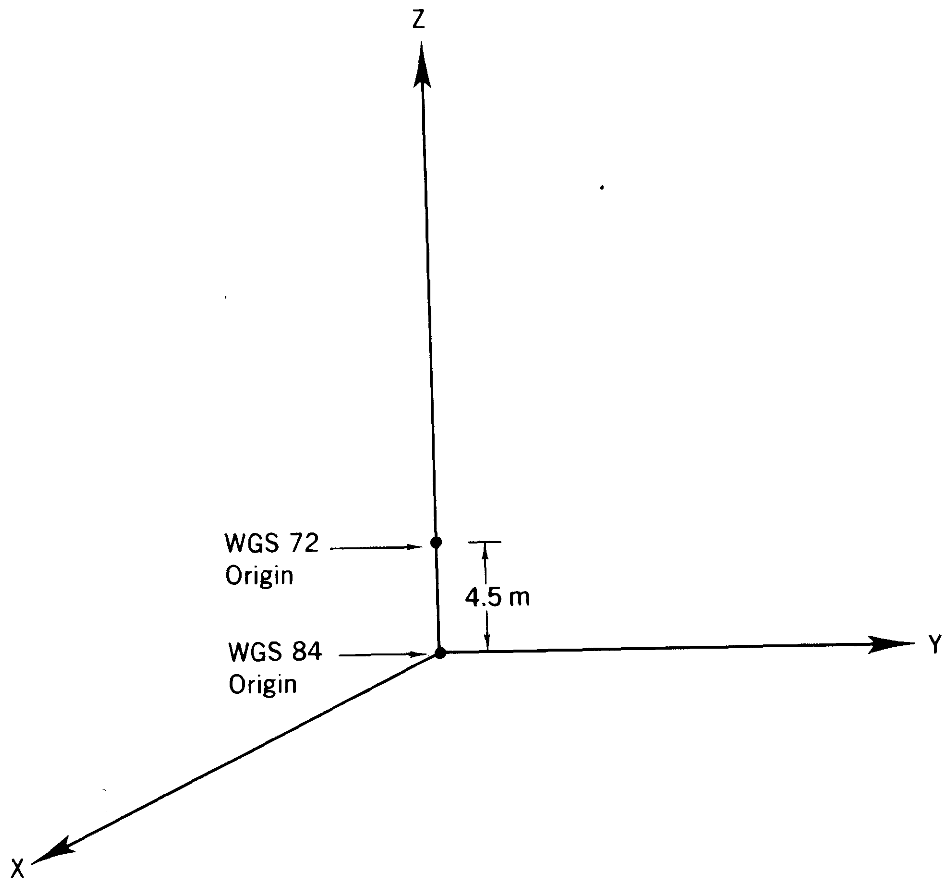


Figure 7.2. Difference Between WGS 72 and WGS 84
Reference Frame Origins

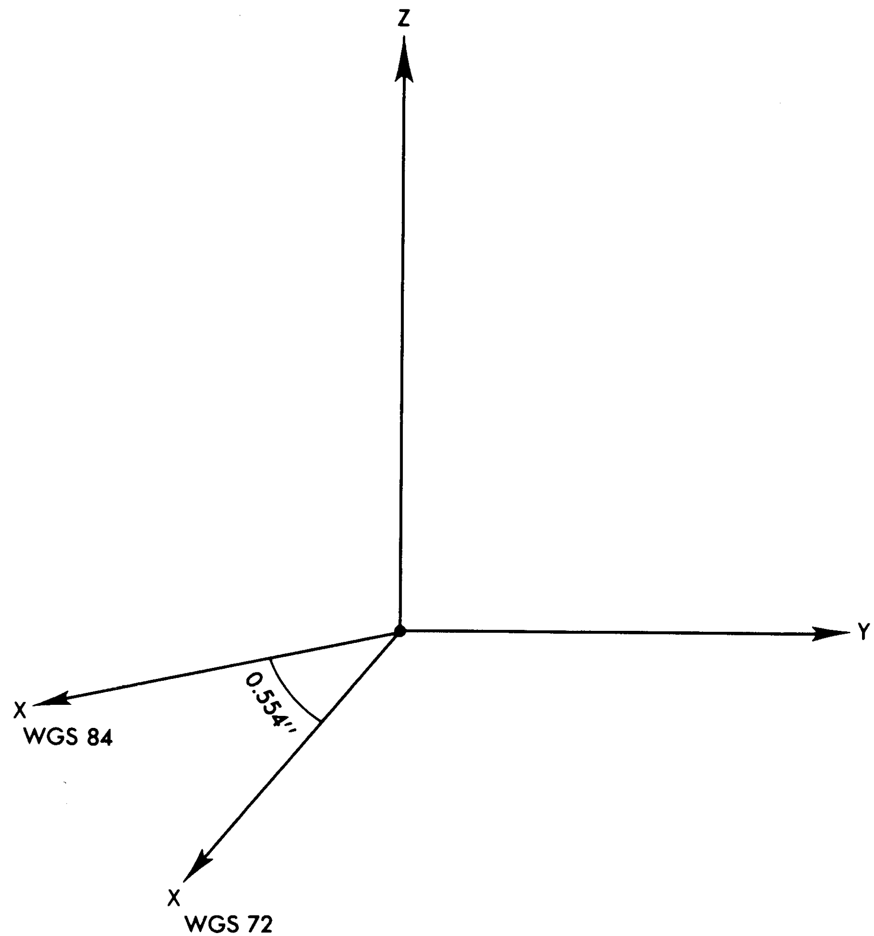
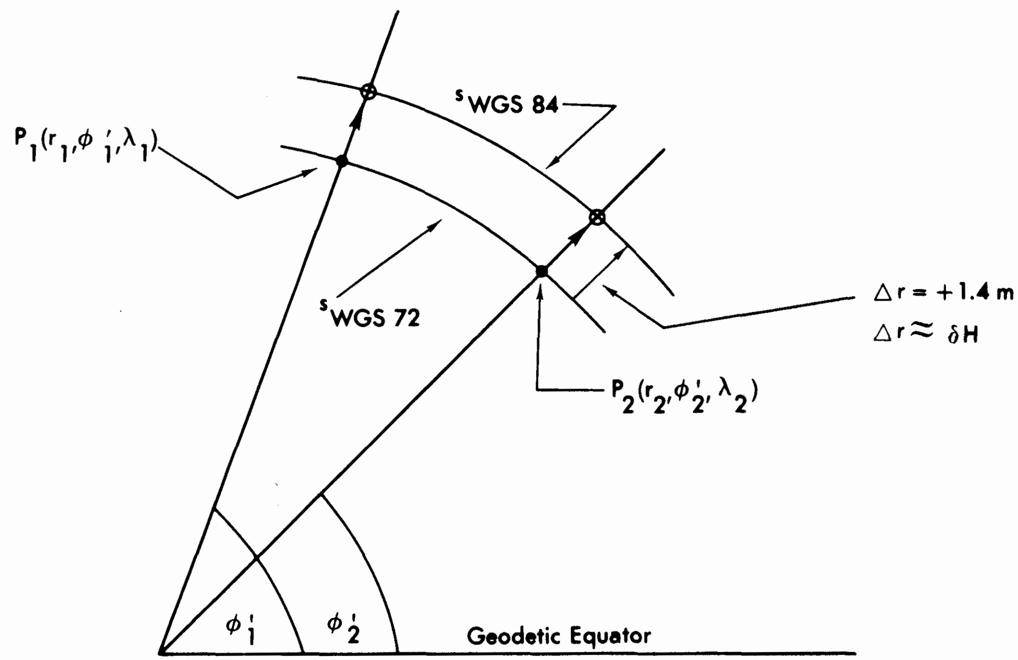


Figure 7.3. Difference Between WGS 72 and WGS 84
Longitude References (X-Axes)

($s_{WGS\ 72} < s_{WGS\ 84}$)

s = distance
r = radius vector
ϕ' = geocentric latitude
λ = geocentric (geodetic) longitude
H = geodetic height



(See Table 7.4.)

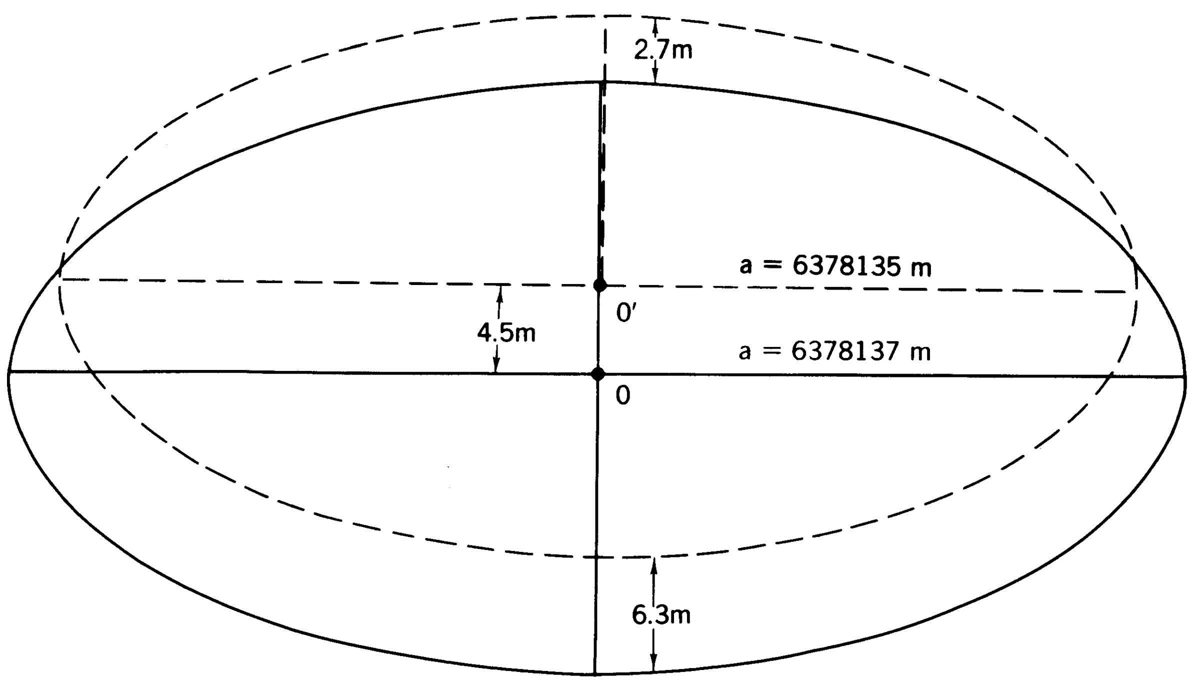
Figure 7.4. Differences Between WGS 84 and WGS 72 Distances and Geodetic Heights (Effect of Scale Change)

WGS 84

a = 6378137m
b = 6356752.3142m
f = 1/298.257223563
(0.00335281066474)

WGS 72

a = 6378135m
b = 6356750.5m
f = 1/298.26
(0.00335277945417)



0 = Origin of WGS 84 Ellipsoid = Earth's Center of Mass

0′ = Origin of WGS 72 Ellipsoid = Earth's Center of Mass (as Known in 1972)

Figure 7.5. WGS 84 and WGS 72 Ellipsoid Differences (Size, Shape)

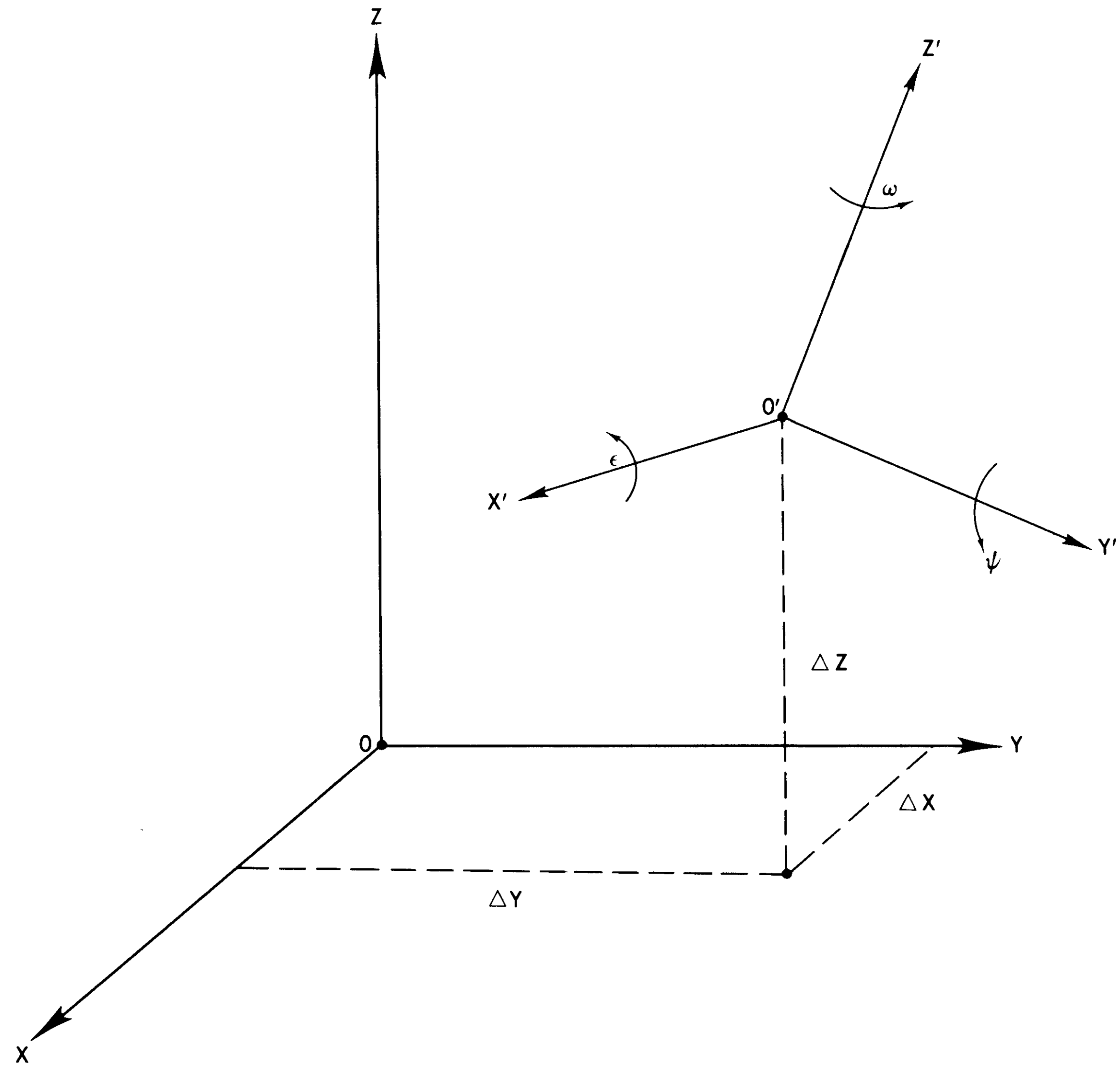


Figure 7.6. Hypothetical Relationship Between Two Coordinate Systems

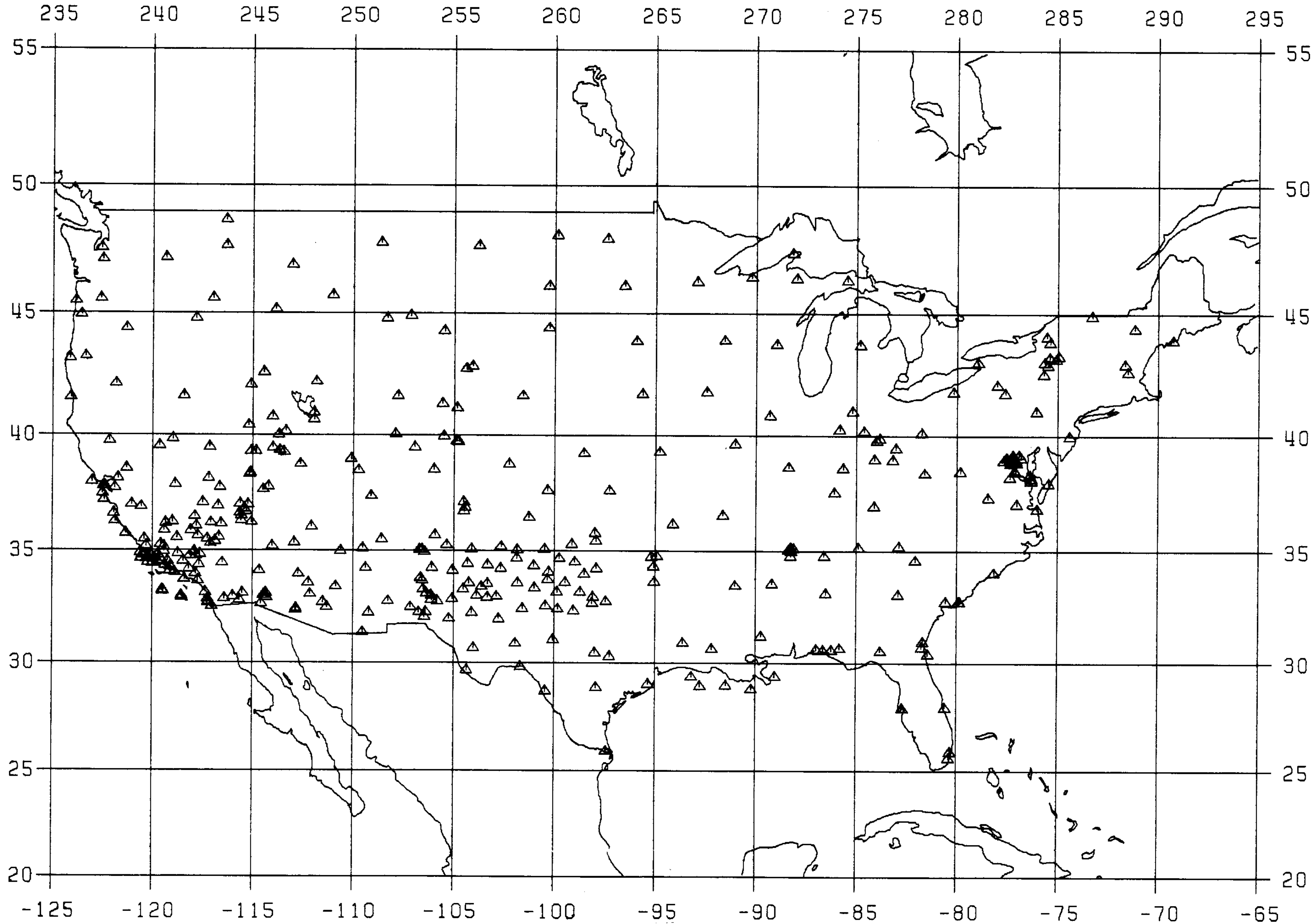


Figure 7.7. Doppler Stations (405) Used in Developing NAD 27 (CONUS) to WGS 84 Datum Shifts

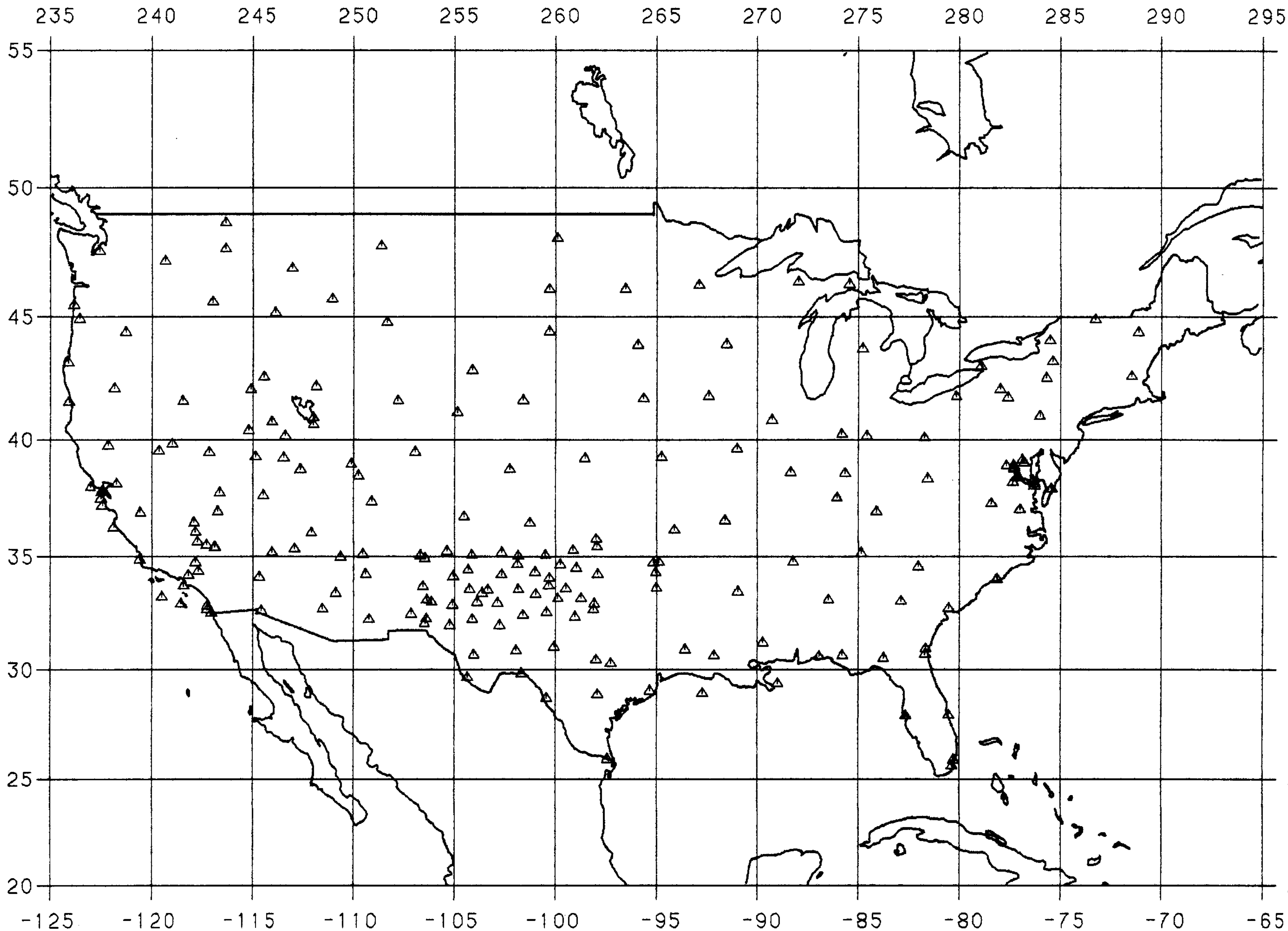


Figure 7.8. Doppler Stations (216) Used in Developing NAD 83 (CONUS) to WGS 84 Datum Shifts

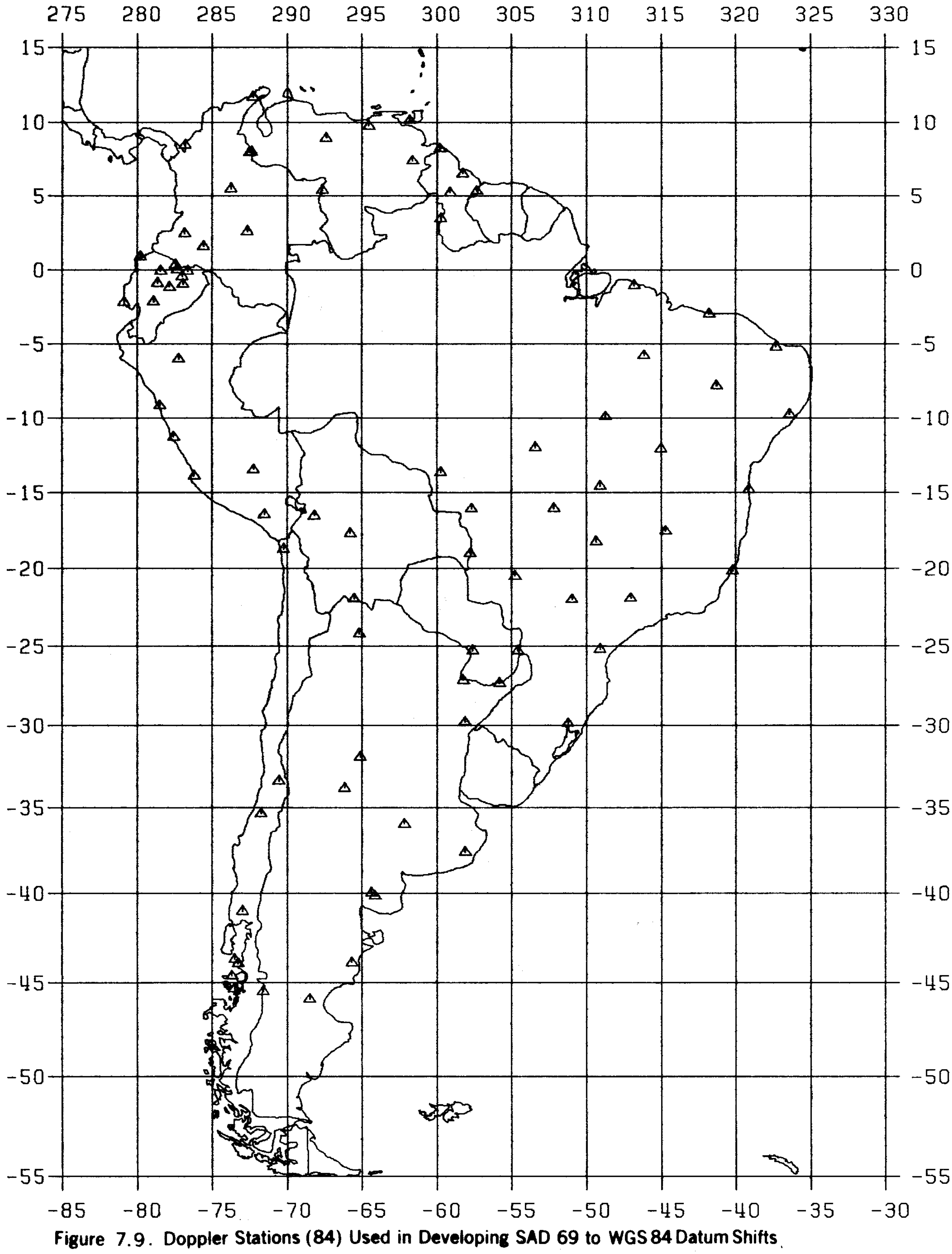


Figure 7.9. Doppler Stations (84) Used in Developing SAD 69 to WGS 84 Datum Shifts.

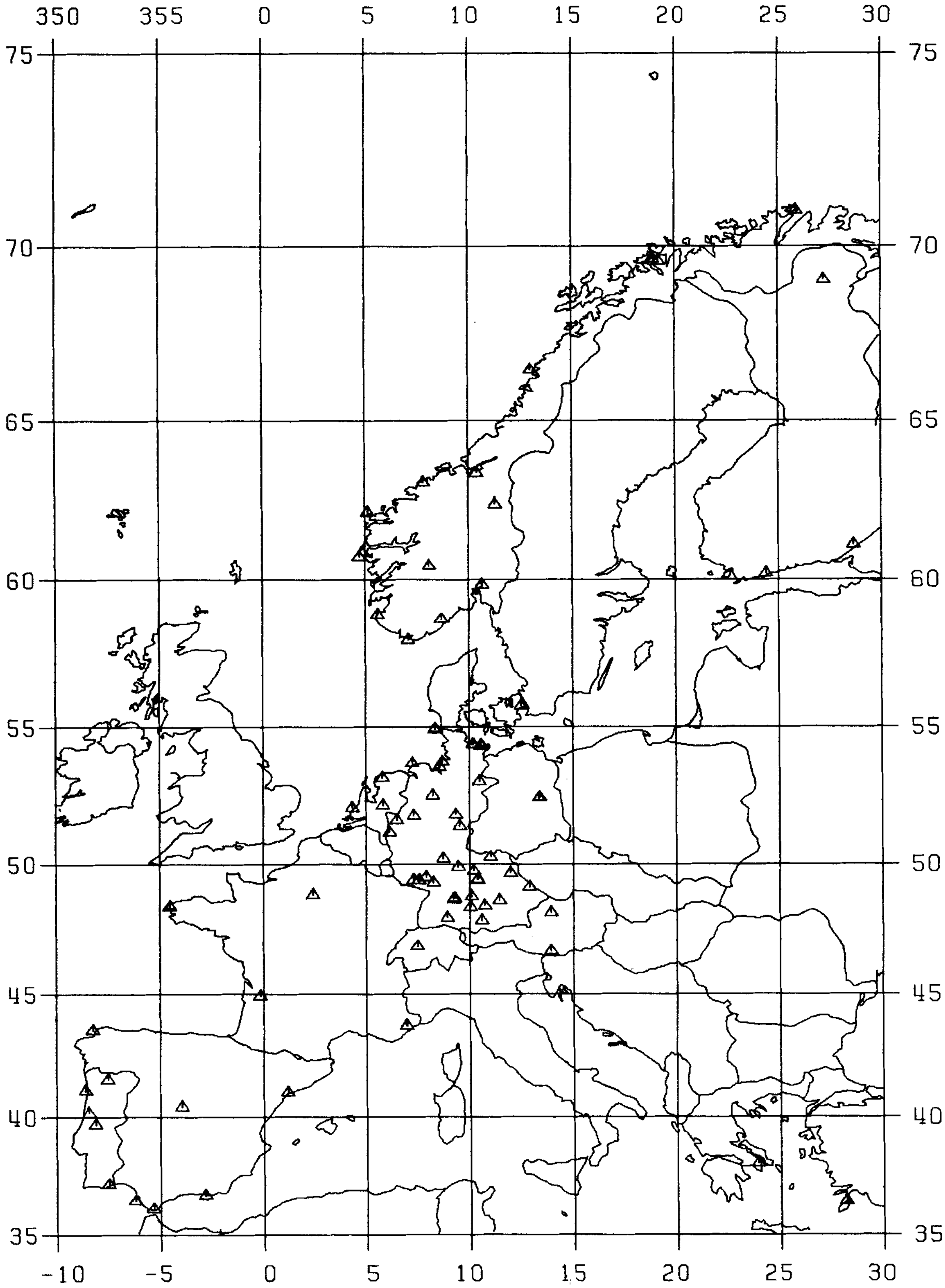


Figure 7.10. Doppler Stations (85) Used in Developing ED 50 (Western Europe) to WGS 84 Datum Shifts

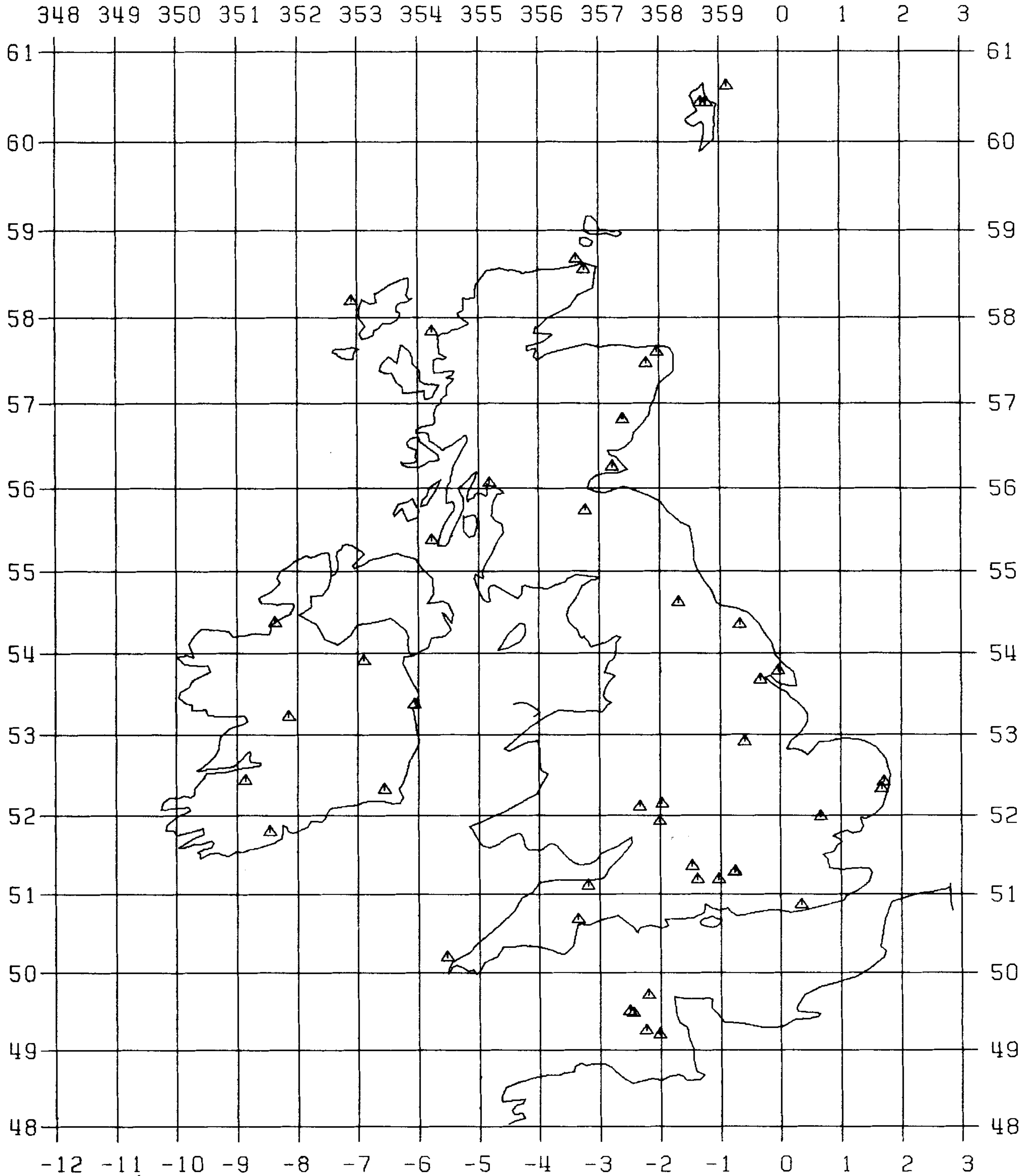


Figure 7.11. Doppler Stations (47) Used in Developing ED 50 (UK/Ireland) to WGS 84 Datum Shifts

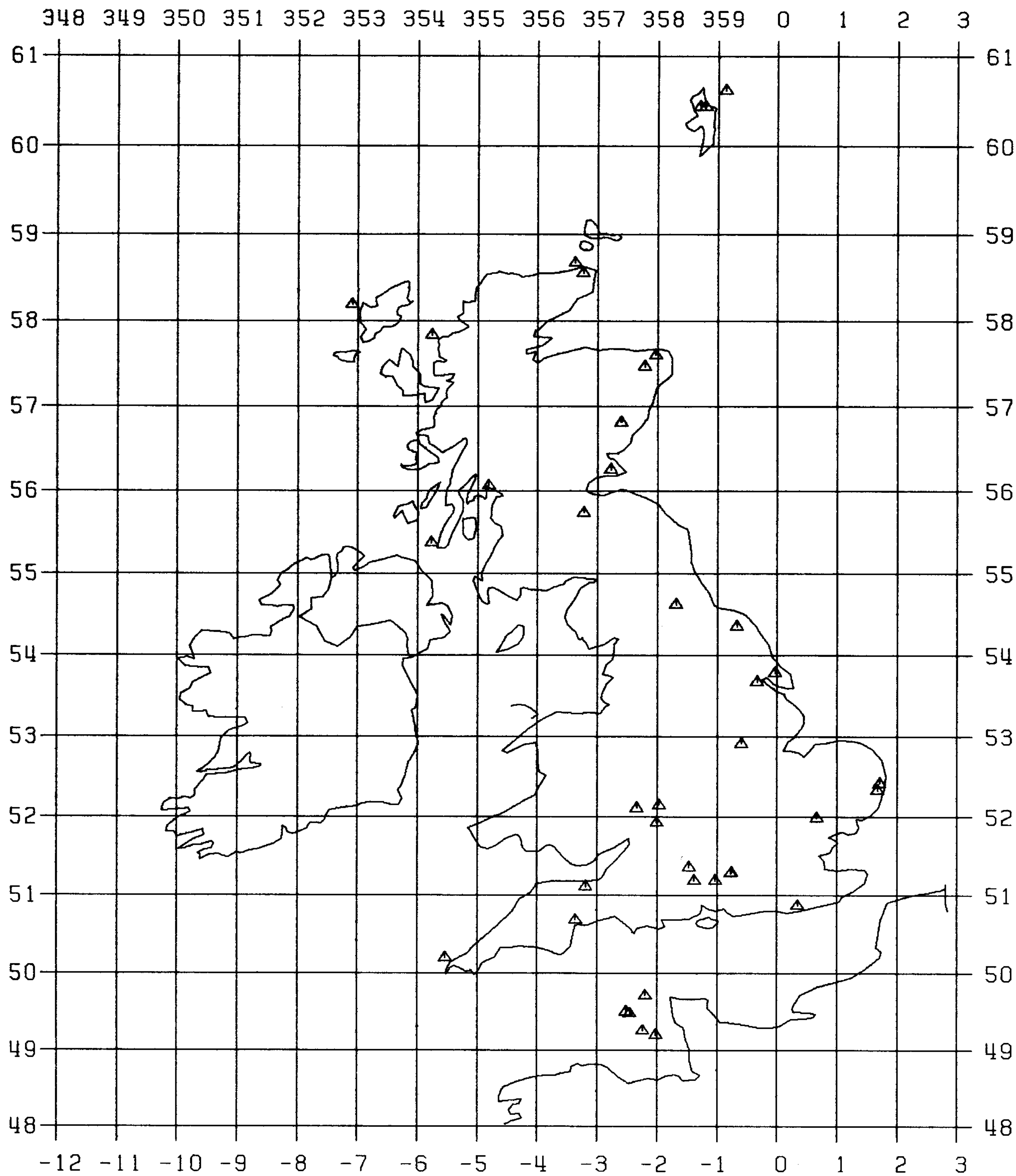


Figure 7.12. Doppler Stations (40) Used in Developing ED 50 (UK Only) to WGS 84 Datum Shifts

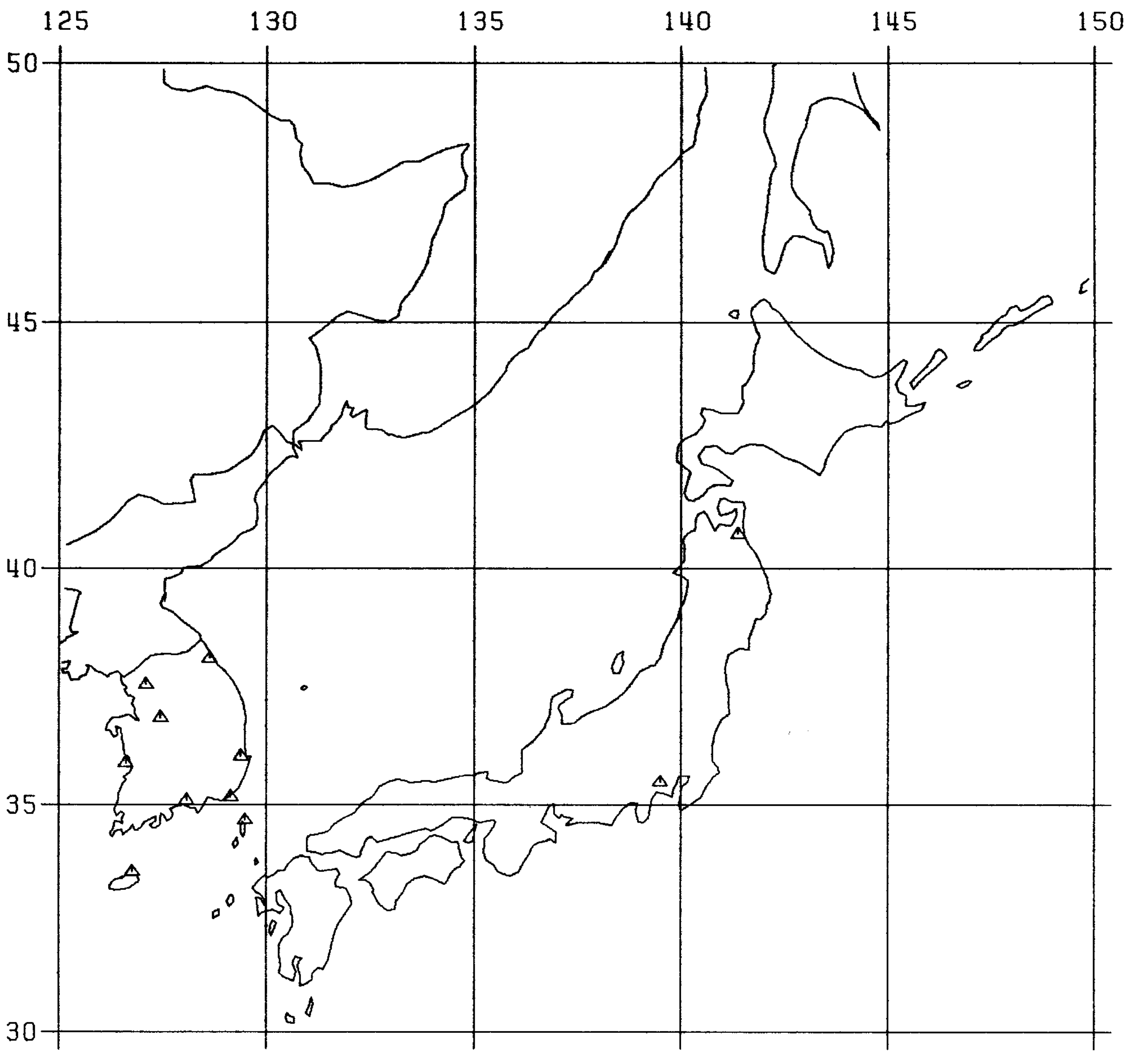


Figure 7.13. Doppler Stations (13) Used in Developing TD to WGS 84 Datum Shifts

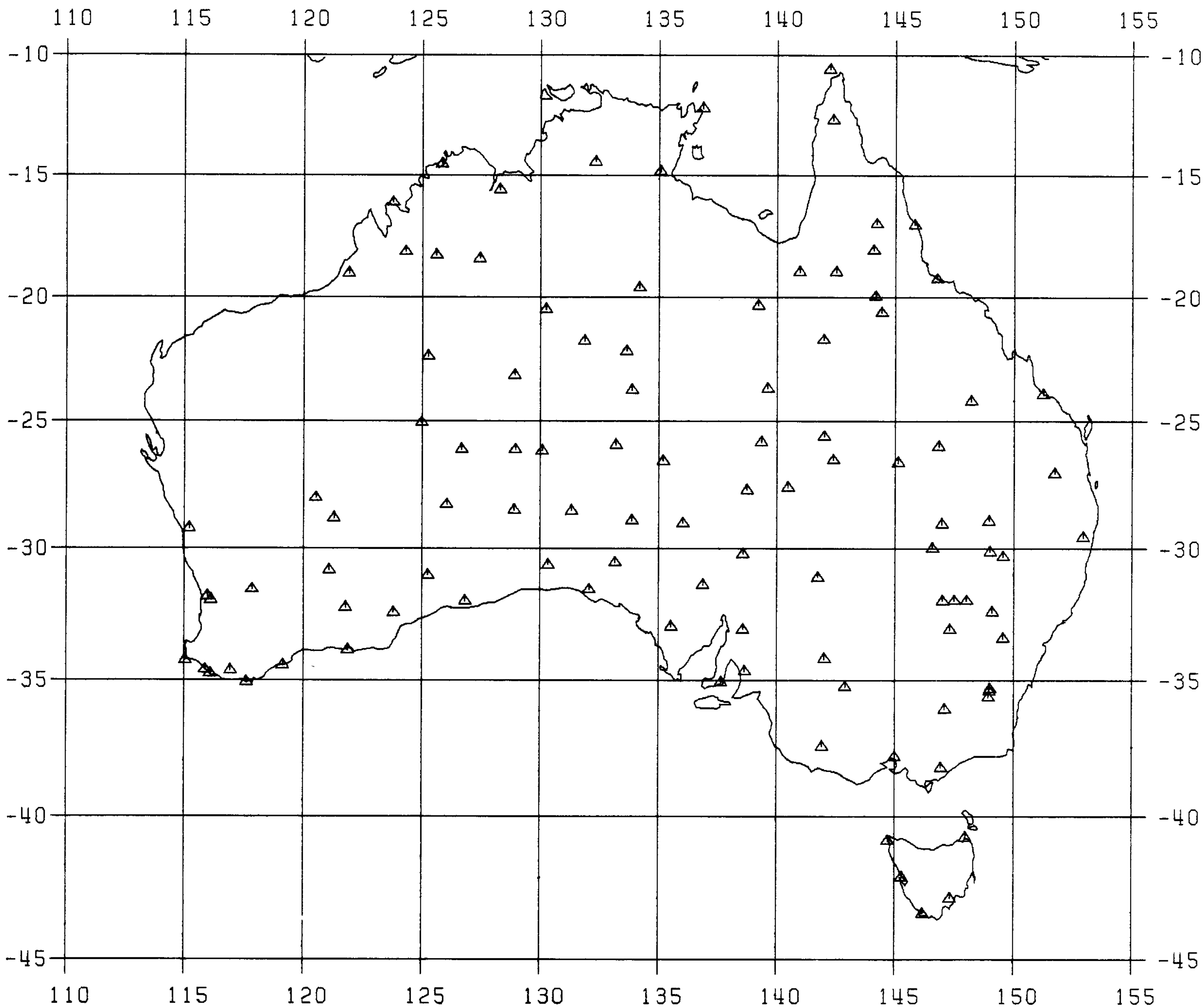


Figure 7.14. Doppler Stations (105) Used in Developing AGD 66 to WGS 84 Datum Shifts

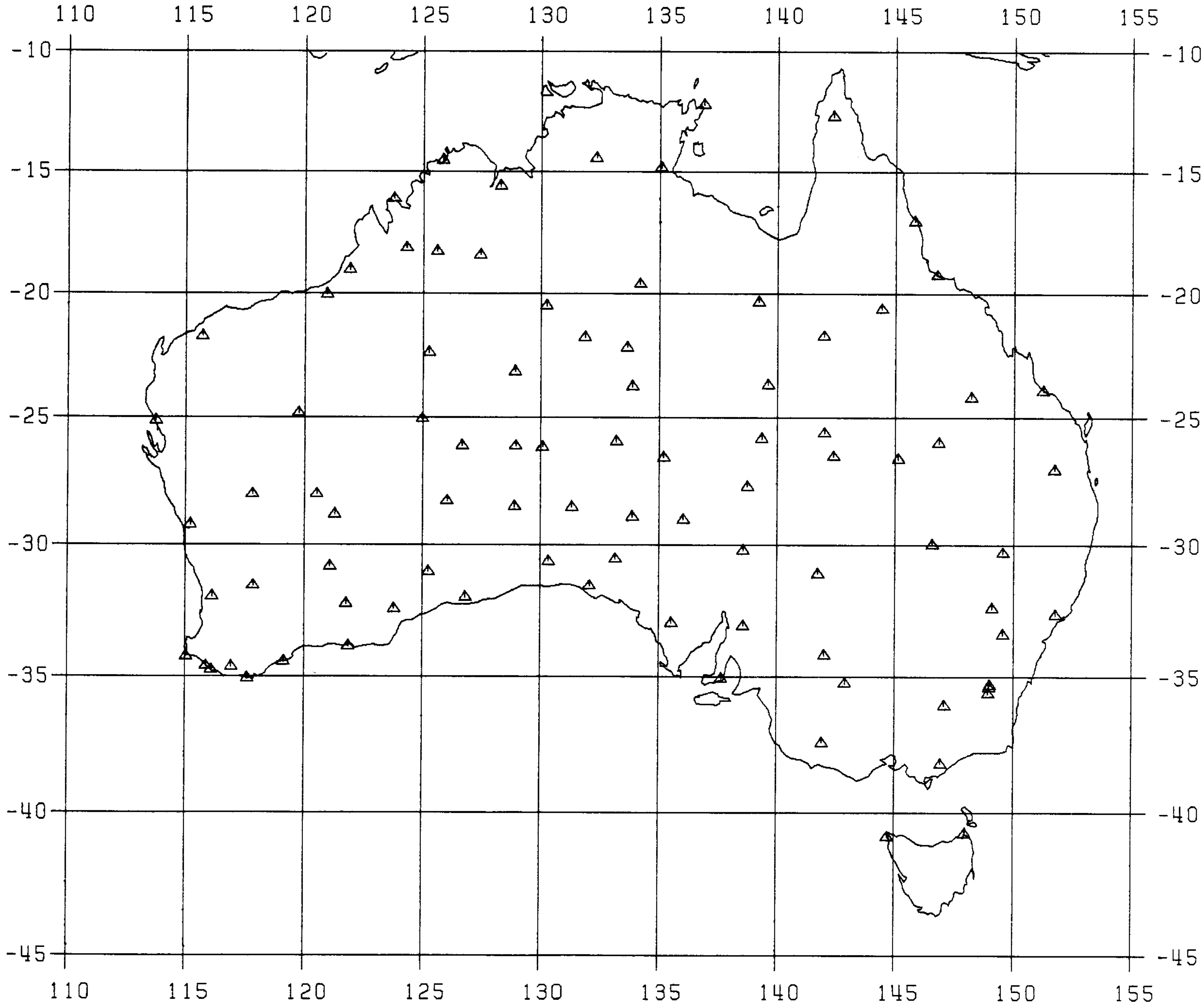


Figure 7.15. Doppler Stations (90) Used in Developing AGD 84 to WGS 84 Datum Shifts

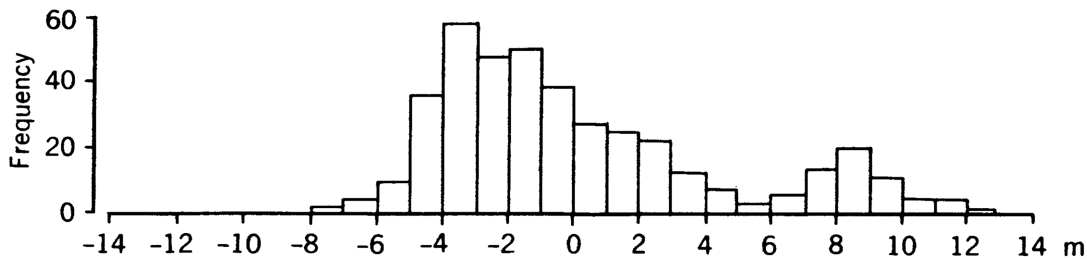


Figure 7.16a. NAD 27 (CONUS)-Geodetic Latitude Differences*

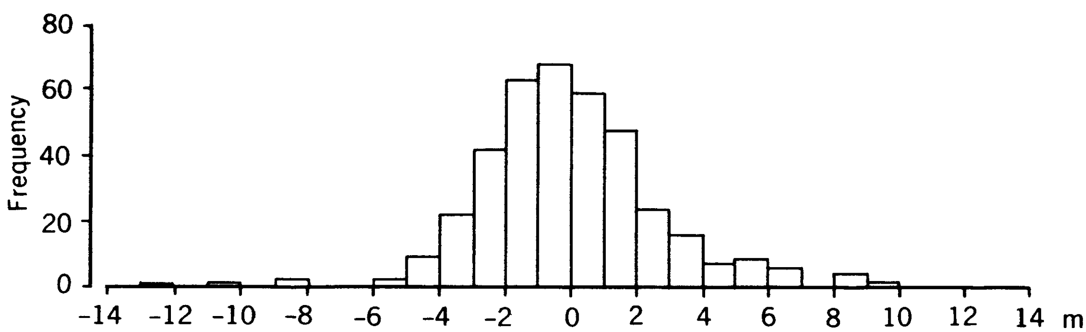


Figure 7.16b. NAD 27 (CONUS)-Geodetic Longitude Differences*

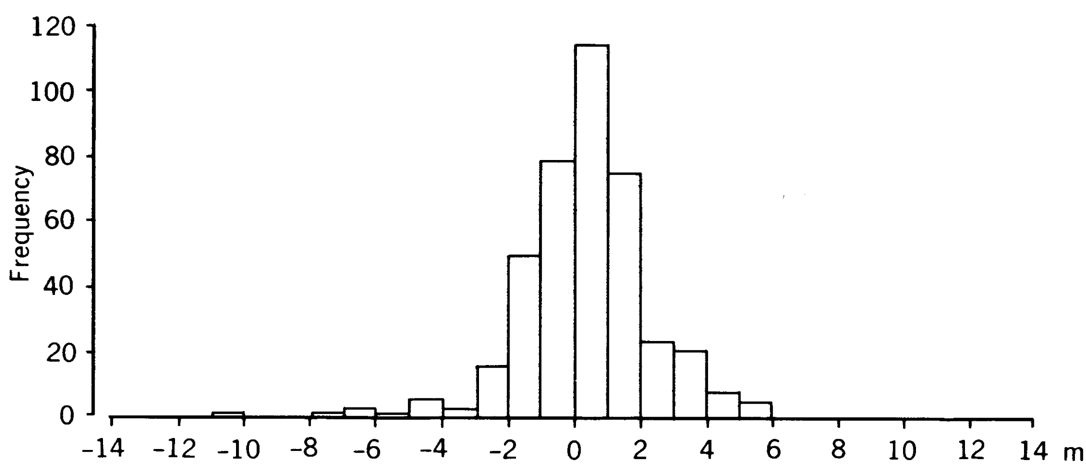


Figure 7.16c. NAD 27 (CONUS)-Geodetic Height Differences*

*Differences = WGS 84 Coordinates Obtained from Doppler-Derived NSWC 9Z-2 Coordinates Minus WGS 84 Coordinates Computed Using NAD 27 (CONUS)-to-WGS 84 Mean Datum Shifts ($\overline{\Delta X}$, $\overline{\Delta Y}$, $\overline{\Delta Z}$) in the Molodensky Datum Transformation Formulas.

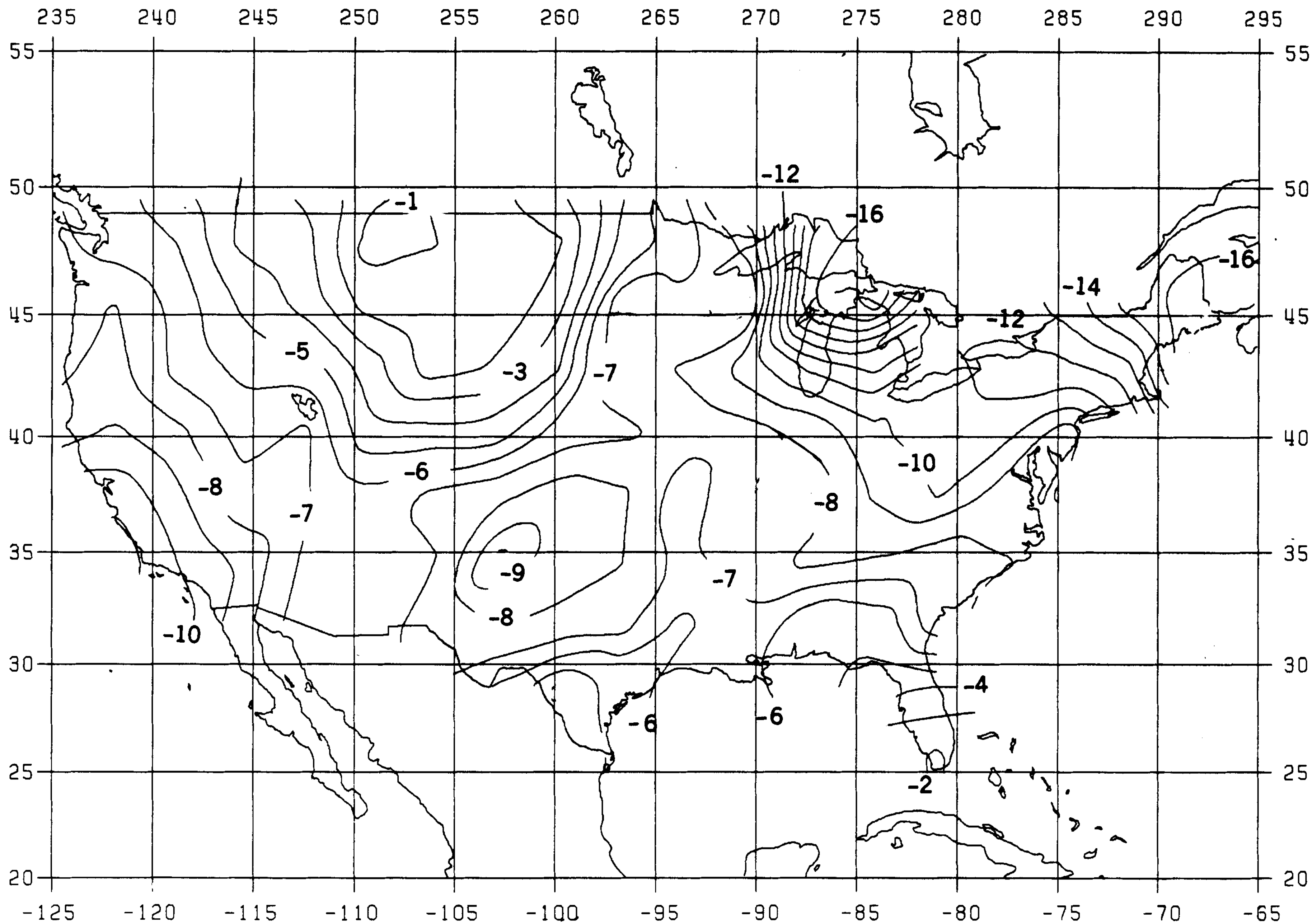


Figure 7.17. Contour Chart - NAD 27 to WGS 84 Datum Shifts (CONUS), ΔX Component (Contour Interval = 1 Meter)

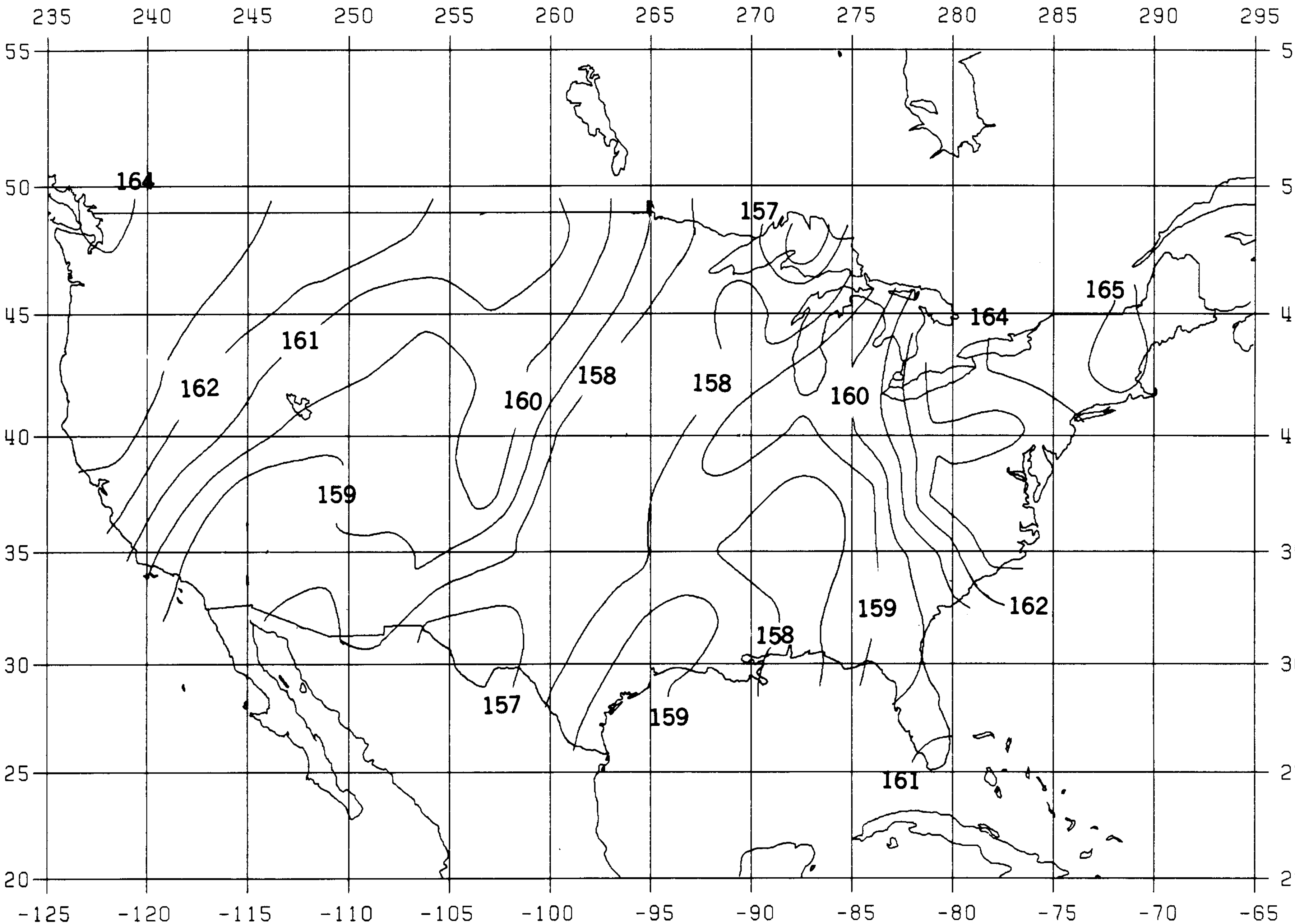


Figure 7.18. Contour Chart-NAD 27 to WGS 84 Datum Shifts (CONUS), ΔY Component (Contour Interval = 1 Meter)

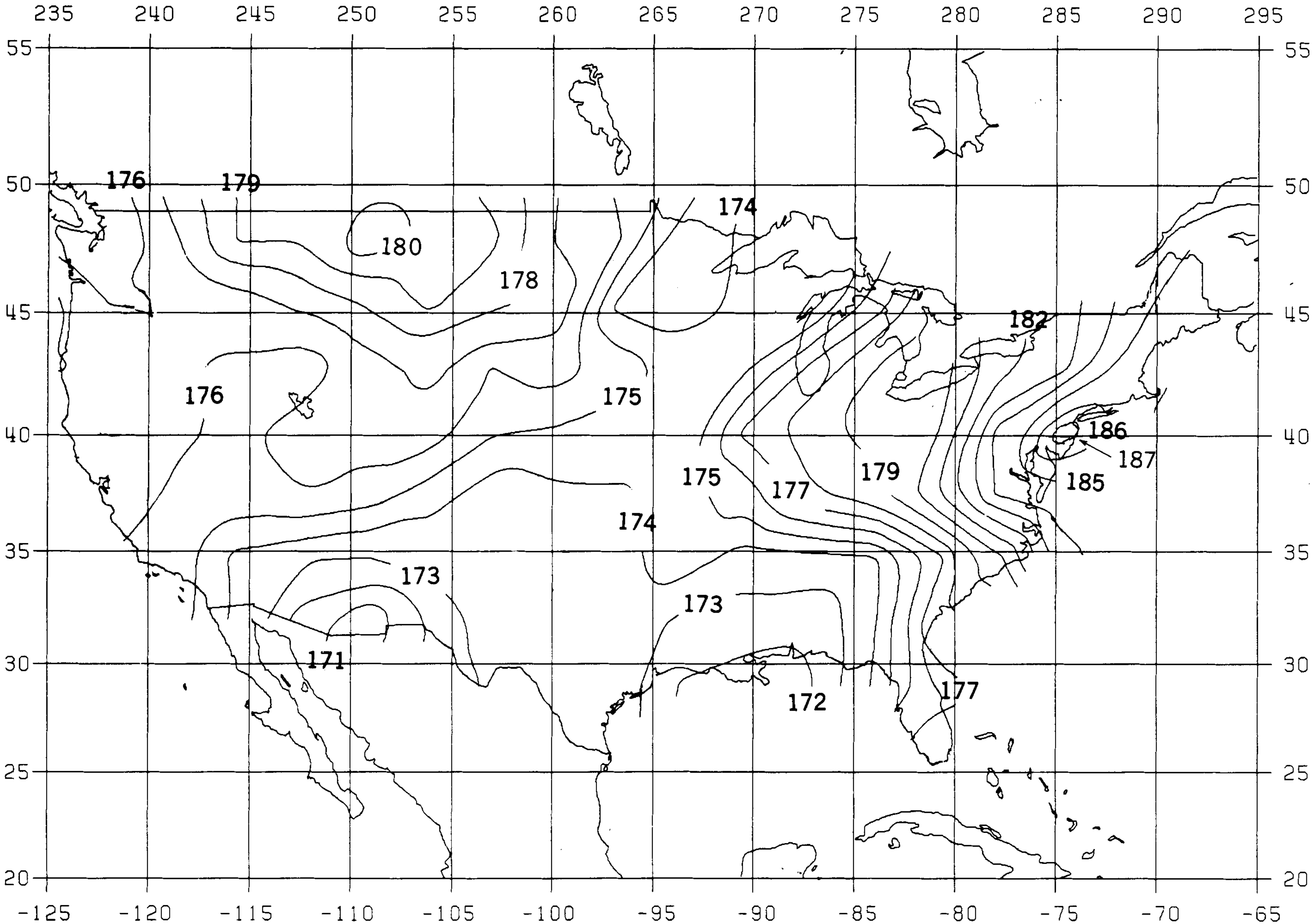


Figure 7.19. Contour Chart-NAD 27 to WGS 84 Datum Shifts (CONUS), ΔZ Component (Contour Interval = 1 Meter)

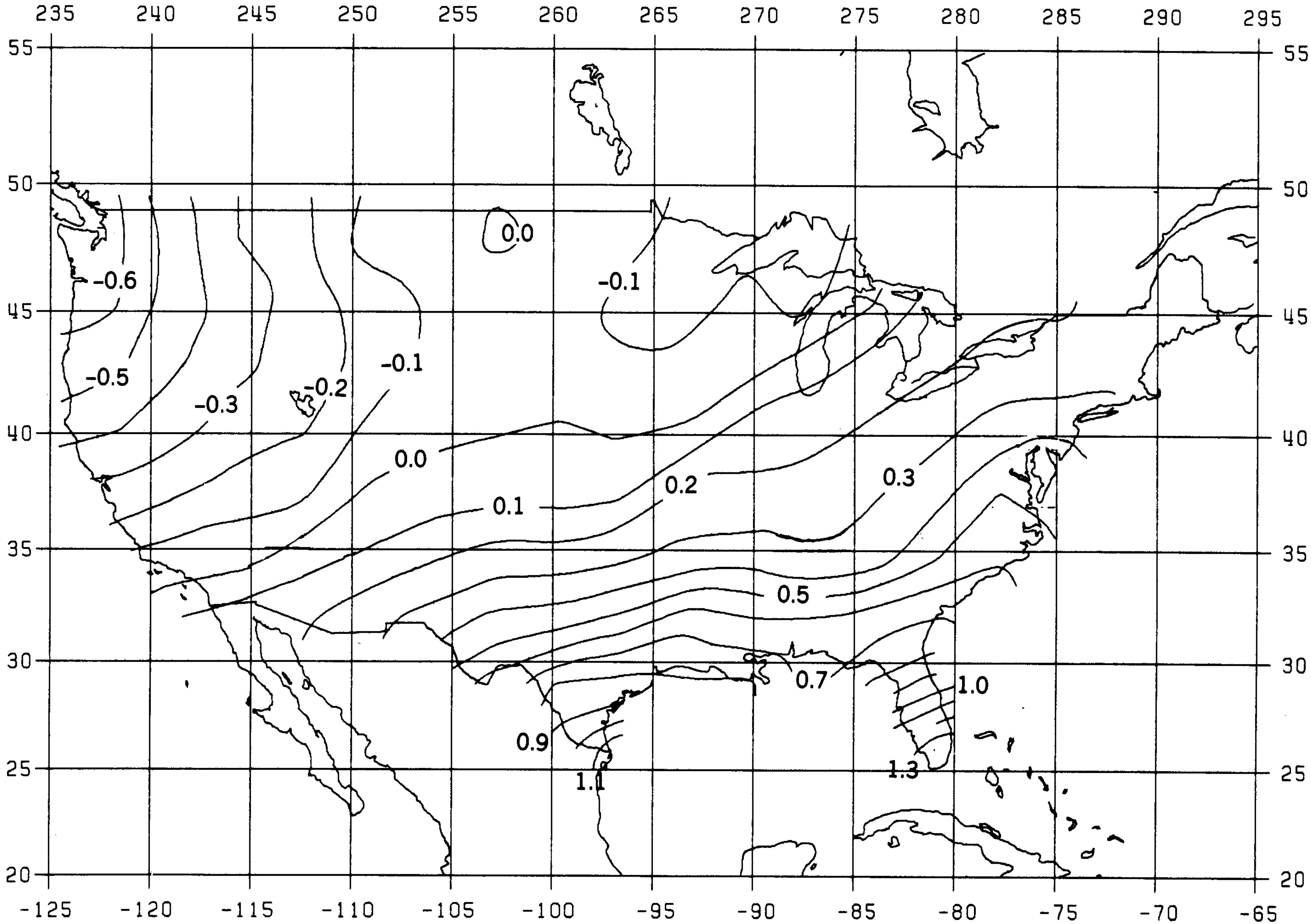


Figure 7.20. **Contour Chart-Latitude Differences, WGS 84 Minus NAD 27 (CONUS; Contour Interval = 0.1 Arc Second)**

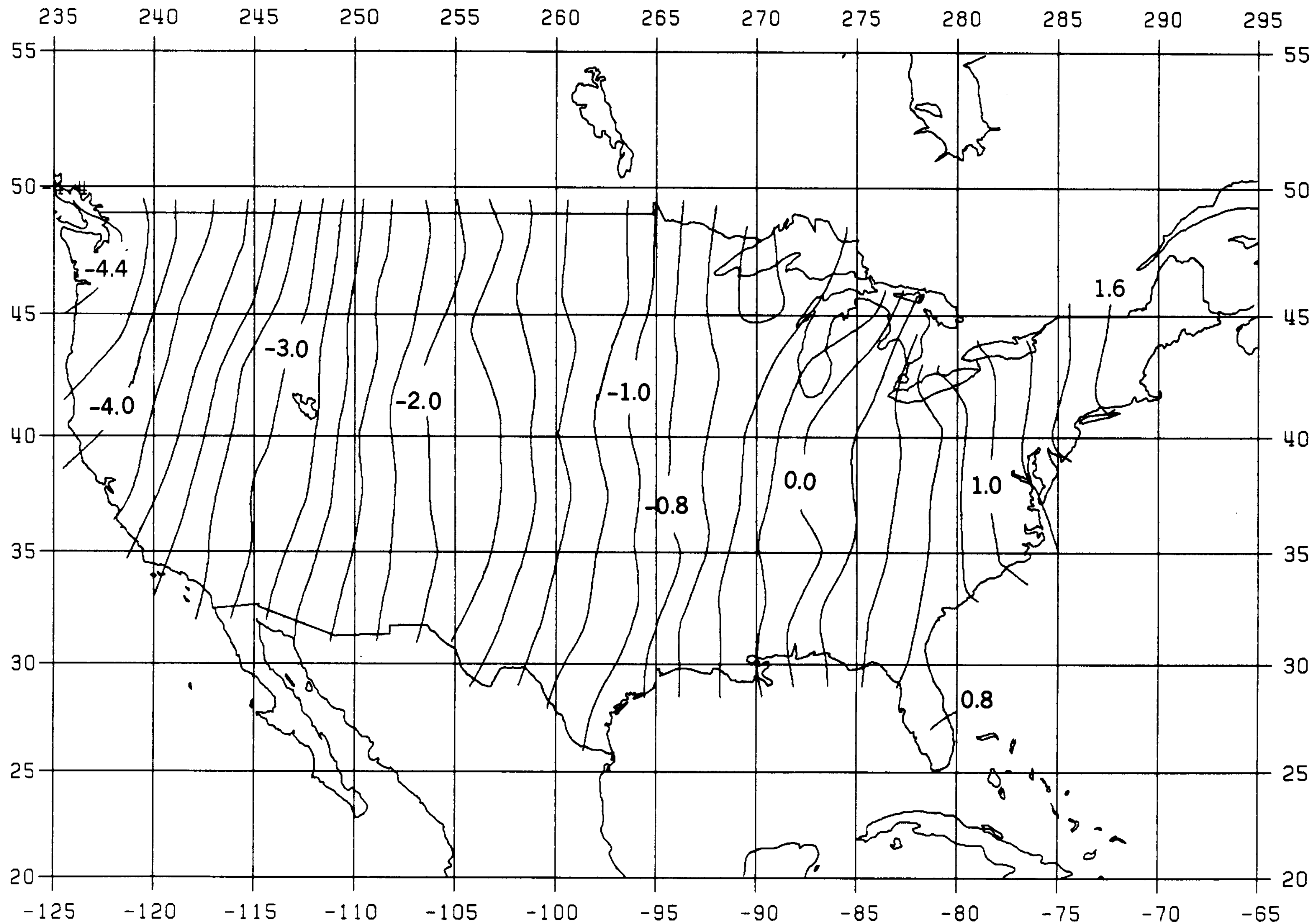


Figure 7.21. Contour Chart-Longitude Differences, WGS 84 Minus NAD 27 (CONUS; Contour Interval = 0.2 Arc Second)

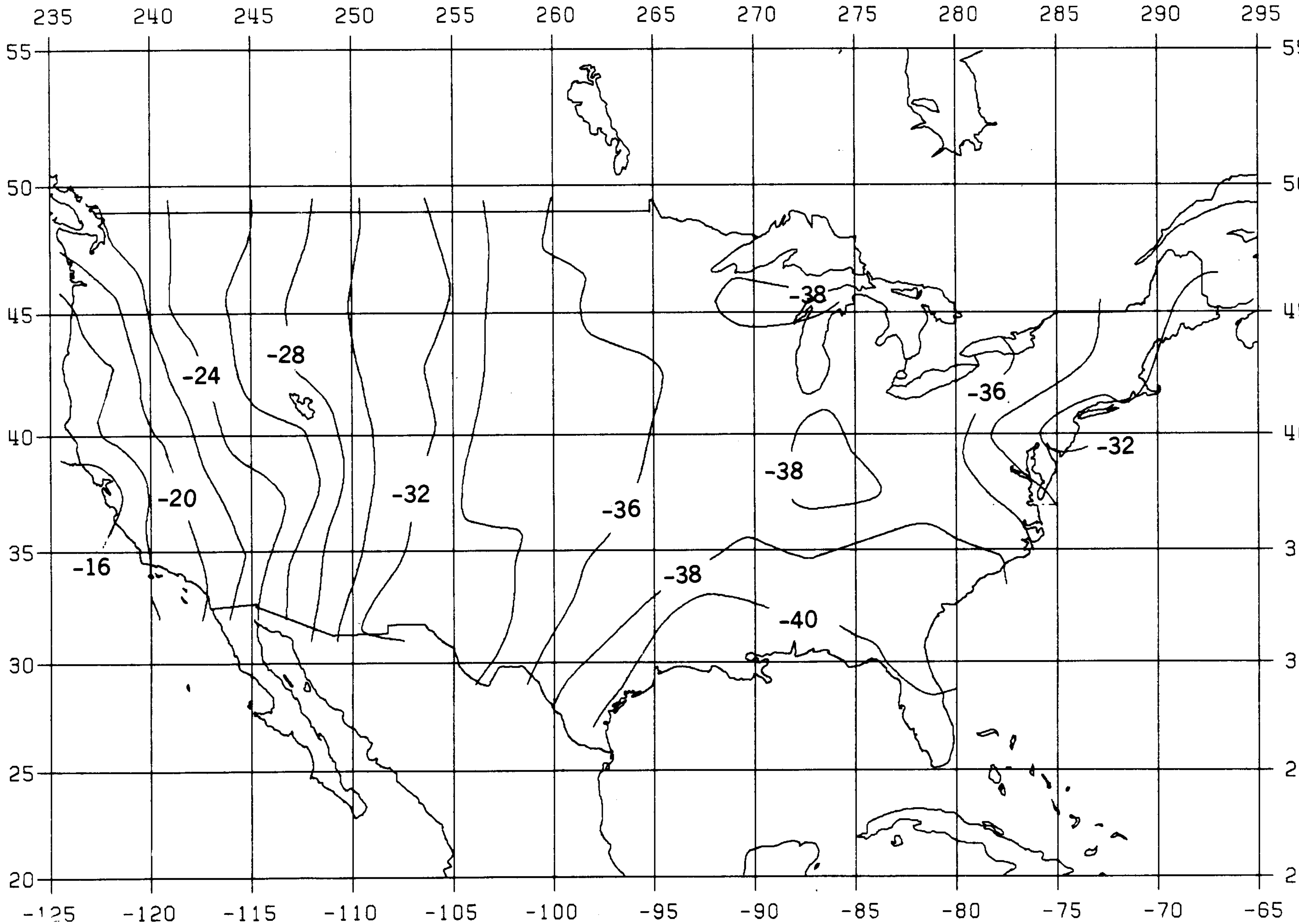


Figure 7.22. Contour Chart-Geodetic Height Differences, WGS 84 Minus NAD 27 (CONUS; Contour Interval = 2 Meters)

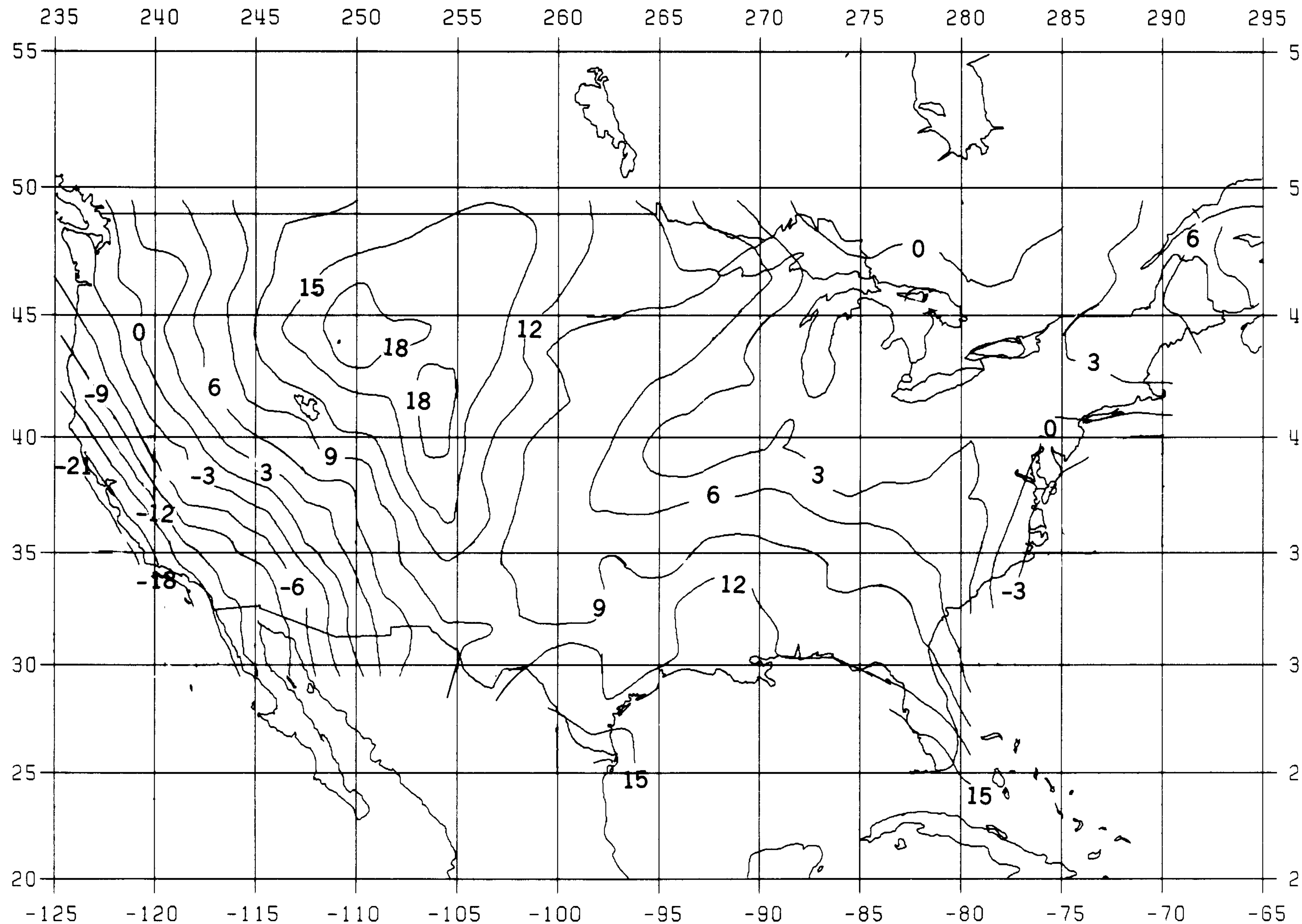


Figure 7.23. DMA-Developed NAD 27 Geoid (CONUS; Referenced to Clarke 1866 Ellipsoid; Units = Meters)

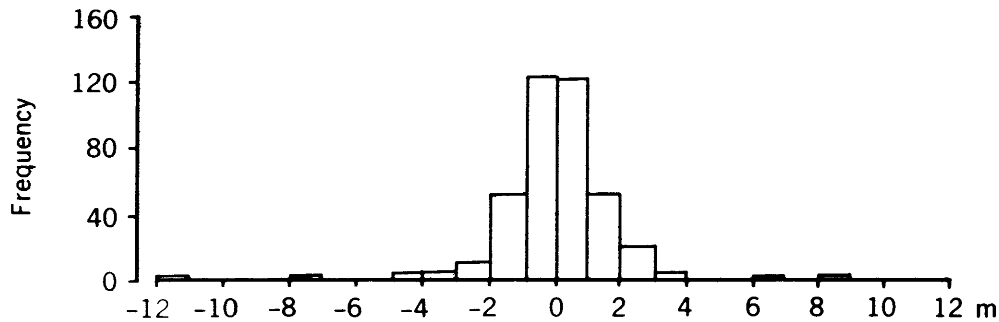


Figure 7.24a. NAD 27 (CONUS)-Geodetic Latitude Differences*

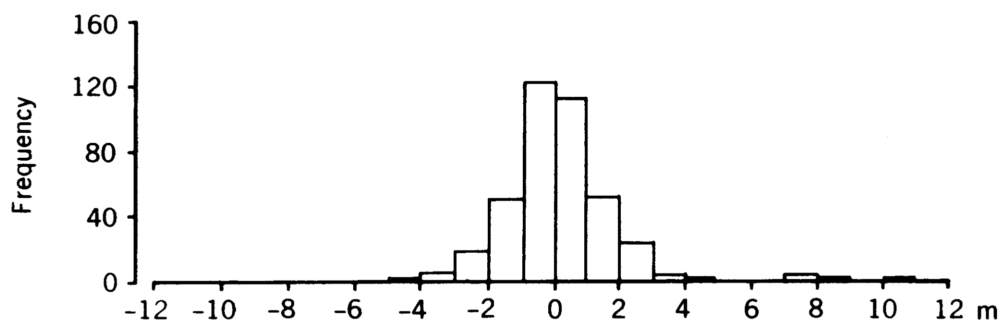


Figure 7.24b. NAD 27 (CONUS)-Geodetic Longitude Differences*

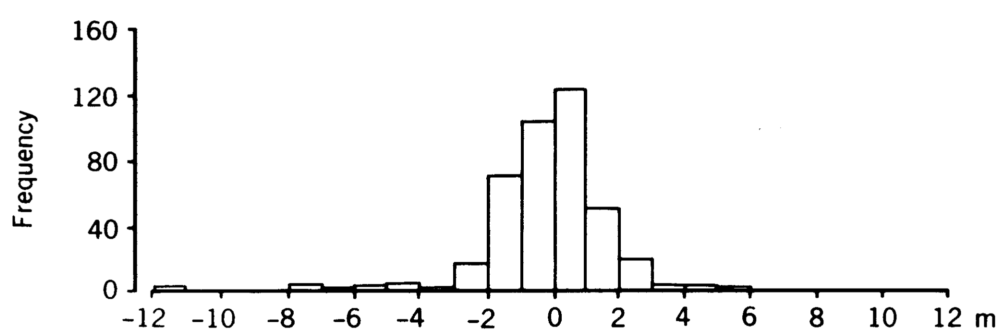


Figure 7.24c. NAD 27 (CONUS)-Geodetic Height Differences*

*Differences = WGS 84 Coordinates Obtained From Doppler-Derived NSWC 9Z-2 Coordinates Minus WGS 84 Coordinates Computed Using NAD 27 (CONUS)-to-WGS 84 Datum Transformation Multiple Regression Equations.