5. WGS 84 EARTH GRAVITATIONAL MODEL (EGM)

5.1 General (Philosophy Underlying the Solution)

The WGS 84 Development Committee decided quite early that the following subset of EGM development objectives could be met by application of the method of least squares in a unified general geodetic solution:

- Determination of the coefficients of a spherical harmonic representation of the gravitational potential to at least the 36th degree and order.

- Recalibration of the systematic errors in the Doppler point positioning system to the one meter precision level.

If successful, WGS 84 would be a significant improvement over the previous system (WGS 72), and an advance in the state of the art of geodesy.

The following (somewhat contradictory) general principles have guided the preparation of both intermediate and final results in the unified solution process:

- The use of a wide and fully representative collection of data, with separate normal equations and evaluation standards for each data type.

- Independent evaluation standards for final evaluation of solutions.

- Accuracy in the computations and in the physical models.

- Statistical treatment of unmodeled physical effects in the formation of normal equations.

- Cost effectiveness in the data exploitation plan, model computations, and evaluations.

Their judicious application should lead to a relatively straightforward solution for model parameters from combined normal equations (after careful development of the equations themselves). In the present EGM development (in contrast to that for the WGS 72 EGM), this hope has largely been realized.

Those familiar with the WGS 72 report will recognize considerable similarity between the EGM development plans for WGS 84 and WGS 72. Nevertheless, the following main differences between the two cover every aspect of the EGM development process:

- Better physical modeling through selection of a larger gravitational potential parameter set.

- Improved equation development, including statistical treatment of unmodeled effects, and the consequential abandonment of empirical weighting of normal equations in the combination solutions.

- Use of newer and more accurate supporting data.

In the following Sections of this report, these features of the WGS 84 EGM development process are discussed in detail.

5.2 Data Sets Used In EGM Development

5.2.1 General

Both the WGS 84 and previous WGS earth gravitational models, as well as several non-DoD models (such as GEM 10), have been principally determined by least squares solutions based on surface mean gravity anomalies and satellite tracking observations. The fact that such

combination solutions have largely replaced the historically independent solutions (based solely on satellite or surface gravity data) must be attributed primarily to their demonstrable superiority, and secondarily to the absence of comprehensive evaluative material unrepresented in the supporting data set with which they might be criticized. The same motivations have led to the consideration for inclusion of satellite radar altimetry and lumped gravitational potential harmonic data in the supporting data set for the present EGM development. Regardless of the reason for inclusion, each of these data resources has a distinct and significant contribution to make, and must be analyzed to define an effective exploitation plan.

Satellite tracking data has been used in earth gravitational model development since the early 1960's. Medium altitude satellites (perigee height of 700 to 1200 kilometers) have been relied upon in this work since they are high enough to be free of large unpredictable atmospheric drag effects and low enough to be sensitive to a significant number of gravitational potential harmonics.

This sensitivity, measured in terms of the resultant periodic (or other detectable) orbital perturbations, is largest for the zonal harmonics (which exert an almost identical force history on each satellite revolution). The so-called primary resonant tesseral harmonics (those which produce a slowly changing or long-periodic force history on each revolution because their order is close to the number of satellite orbital revolutions in one day) excite the next largest effects. These are followed by the harmonics of the lowest non-resonant orders, secondary resonant harmonics (order close to the number of revolutions in two days), and other non-resonant orders. Considering nominal sized harmonic coefficients, the size of the corresponding orbital perturbations varies from hundreds of meters for the principal contributors to much less than a meter for the high-order non-resonant tesseral harmonics. The exploitation of extremely accurate tracking data is necessary if these small sub-meter periodic orbital effects are to be resolved.

The first requirement of a satellite data exploitation plan is a definition of the information content of satellite orbital histories. The first consideration is that the number of passes, and not the generally much larger number of processed observations, limits the information reducible from the data. Effectively, each pass contributes only two observations, and, because of the correlation of model errors, little new information is obtained from additional passes close to those already processed. Also, the similarity of orbital perturbations of harmonics of the same order for a given satellite (inclination) further limits the reducible information content to four determinate (linear combinations of) harmonics of each order being solved for. For a solution to the 30th order (assuming such a model can be supported by satellite data alone), each satellite could contribute at most 244 effective observations.

To extract this information, some investigators [5.1] have advocated observing each satellite throughout a full revolution of perigee, although theory and experience with modern Doppler satellite data show that processing observations at two distinctly different perigee conditions is sufficient. (If observing at only one perigee condition, up to half of the above information may be lost.)

To obtain the necessary number of passes, only a few days of Doppler tracking is needed, since from 50 to 200 passes per day are typical during observation campaigns. However, two six-day spans were recommended to ensure tracking at least one full cycle (which is two days or longer) of the primary resonant perturbations at each perigee condition. Because of the relatively lower density of laser passes (typically eight per day during campaigns), it was thought necessary to process much longer spans (60 to 120 days) to fully extract the basic orbital information using this data type.

To the extent that additional data for the same satellite (inclination) is processed beyond the above minimum sufficiency point, only additional strength of determination of the same basic orbital

information is obtained. To obtain information on different (linear combinations of) harmonics, it is necessary to process tracking data for distinctly different (inclinations of) satellites, or to utilize non-satellite data.

The other principal data contributor (and, in fact, the main support of the present solutions) is surface mean gravity anomaly data. Mean gravity anomaly data has also been used historically in earth gravitational model development since the 1960's, although not always in conjunction with the method of least squares. Of particular interest in this regard are the WGS 66 mean gravity anomaly-based EGM [5.2], the OSU 81 EGM [5.3], and the independent orthogonal integration solutions to be presented later in this Chapter.

Aggregation of the point gravity measurements to equal-area means is an effective way to reduce the very large number of available data to a tractable and geographically balanced supporting data set for earth gravitational model development. At the same time, averaging minimizes the corrupting influence of unmodeled higher degree gravitational harmonics according to Pellinen's area averaging factors [5.4]. Averaging, in practice, is performed using the worldwide 1°x1° equiangular mean gravity anomaly set as the basic data.

In the exploitation plan for surface data, the grid spacing and averaging area need to be defined. It is customary to equate the grid spacing and area size so as to obtain the maximum number of independent aggregated means (without correlation due to common included 1°x1° data). The area size must be selected by trading off the relatively greater number of observations (effectively information, while this number is small) against the relatively lesser strength of each observation (according to the increase in the unmodeled error variance) as the averaging area decreases. The use of 3°x3° area means extracts gravitational model information from the basic data which is sufficient to determine harmonics out to the 60th degree without aliasing due to (nearly) uniform grid spacing. (Aliasing would have been possible at the

36th degree using the more conservative 5°x5° means selected in other EGM developments.)

Because of the widely varying accuracies and possible inter-observational correlations with some observational types contributing to the basic gravity anomaly data set, it is also necessary to consider which types will be admitted to the determination of the equal-area means. In this exercise, a rather broadly inclusive philosophy has been followed. In particular, mean gravity anomalies determined by transformation of GEOS-3 and SEASAT-1 geoid height data [5.5] - [5.7] have been used wherever the more conventional surveying or extrapolation methods are of lower accuracy. This decision results in a substantial improvement of the mean gravity anomaly data set over the oceans. Furthermore, it would (essentially) not be a dual inclusion of data area if 3°x3° altimetric mean geoid heights were also to contribute to a solution, since the geoid height to gravity anomaly transformation relies heavily on very high frequency information not contained in the 3°x3° mean geoid heights.

Principles for the inclusion of altimetric mean geoid heights are quite similar to those just enunciated in the discussion of mean gravity anomalies. However, it is important to note that while mean gravity anomalies contribute strongly to the determination of all the harmonics, the altimetric mean geoid heights serve mainly to define the long and intermediate wavelengths of the earth's gravitational field. Under these conditions, the averaging area size is less critical and was set to 3°x3° to maintain consistency with the surface element size used for forming the mean gravity anomaly data.

Two other data contributors which are new to WGS EGM development, namely tracking data for the NAVSTAR GPS and LAGEOS Satellites, serve mainly to define the harmonic coefficients of low degree and order. In the case of the very high altitude GPS satellites, the contribution is almost exclusively limited to the resonant even order harmonics. With this exception, the same analysis applies to these higher

altitude satellites as was given earlier for medium altitude satellites. However, relatively longer data spans must be processed to effectively extract the orbital information content.

It is worth noting that regardless of the small number of effective observations obtained by processing data from these last two contributors, inclusion of their data in the proposed solutions will enhance the usefulness of the resulting EGM in serving the contributing systems (for instance, by improving the accuracy of NAVSTAR GPS orbits), and, generally, for all users through more correct harmonic coefficient values.

Finally, outside the realm of precise orbital analysis, which is the province of only a handful of specially instrumented satellites, there is an extensive collection of tracking data of lower accuracy [such as that collected by Naval Space Surveillance (NAVSPASUR), the North American Air Defense Command, etc.] on literally thousands of satellites. If gravitational harmonic perturbations are sufficiently large, the harmonics responsible may still be estimable with this data. In practice, this requirement is met generally for zonal harmonics, and occasionally (in deep resonance conditions) for some tesseral harmonics.

Orbital decay due to atmospheric drag causes the period of medium altitude satellites to decrease until reentry (and disintegration) occurs. While the orbital frequency is very near m revolutions per day, the satellite is in deep resonance with the order m harmonics. Generally, two (but occasionally as many as 10) effective observations of (linear combinations of) the harmonics are reducible from an analysis of each resonance passage. As before, it is desirable to collect data at different inclinations.

A mixture of numerical methods and the general theory of orbital perturbations has been used by several investigators, notably King-Hele [5.8] - [5.13] and Wagner [5.14] - [5.19], to analyze the available data. The work has been in progress for approximately 20 years.

Full exploitation, requiring analysis of 10 to 20 satellites in resonance with each order, has been accomplished for orders 14 and 15, where there is a relatively greater abundance of data, and for the odd zonal harmonics. Some data is available from satellites in resonance with other orders also. The resulting effective observation equations for the values of the linear combinations of harmonics (called lumped coefficients) are available in the publications of these investigators.

5.2.2 Doppler

According to basic physics, the Doppler shift, a change in the apparent frequency of a moving frequency source, is proportional to the time derivative of the range (or distance) between source and receiver. Practical Doppler observations are obtained by integrating over short time intervals, effectively converting range rates to range differences. (Actual integration consists of counting full cycles of the slowly varying phase difference between the received satellite frequency and a nearby reference frequency available in the receiver.) Two coherent Doppler signals at different reference frequencies can be used to compensate unpredictable (but frequency-dependent) ionospheric propagation effects. Tropospheric effects must be compensated by modeling.

Modern tracking equipment consists of a pair of rudimentary fixed antennas and the advanced electronics required to measure the time intervals and Doppler counts with high precision. Both the tracking receiver and the satellite transmitter must have very stable frequency sources for geodetic quality measurements. It is also desirable for the satellites to be stabilized in their orbits and for the transmitting antennas to occupy fixed positions in the satellite local-vertical coordinate systems.

Doppler tracking stations have been operated by the DoD and by cooperating scientific agencies worldwide since the early 1960's. In the historical period (see below), the Doppler network consisted of 13 fixed and seven mobile receivers. More recently, the station network has included as many as 20 fixed and 20 mobile receivers operating simultaneously. The mobile receivers, which have, over time, been located at hundreds of sites worldwide, were designed for surveying applications. For a high priority satellite, each station can collect four or more passes (depending on latitudes) of Doppler data each day, for a total that may exceed 200 passes per day.

Significant improvements in the tracking network were made in 1971 when the collection of continuous count (instead of sampled) Doppler data was begun and, in 1975, when ultrastable rubidium oscillators were installed in the fixed receivers. The year 1971 roughly separates the historical and the modern period of tracking data. Modern tracking data is significantly more precise than historical data for geodetic applications.

Databases of collected Doppler tracking data are maintained by NSWC and DMAHTC. Although data on approximately 50 satellites is available, the vast majority was collected on satellites at 90° inclination or on satellites unsuitable for EGM development. Useful data is available for satellites at five distinct inclinations in the modern period, and six in the historical period.

Doppler data on seven satellites, including all five modern and two of the historical satellites, was included in the development of the WGS 84 EGM. The common names and inclinations of these satellites, as well as other orbital characteristics, are presented in Table 5.1. It is regrettable that only partial exploitation of the High Latitude (HILAT) Satellite [5.20] data for WGS 84 EGM development was possible due to time constraints. The original exploitation plan was followed for each of the other Doppler geodetic satellites.

For each of the seven satellites of interest, two six-day data spans (Arcs A and B) were selected for processing which satisfied, as much as possible, the need for dense Doppler tracking data, good balance between northern and southern data acquisition stations, significant variation in argument of perigee, and absence of intractable peculiarities. In some cases, the possibilities for selection were fairly limited due to short observation campaigns, tracking priority conflicts with other satellites, satellite equipment failures, the influence of bad weather or tracking performance, and other uncontrollable circumstances.

Satellite data processing is rather arbitrarily divided into work called preprocessing, which can be performed without the aid of a good satellite orbit, and orbital analysis (or just processing). Most of this work, prior to the stage of development of the full normal equations, is performed using one of the many versions of the NSWC computer program CELEST [5.21], which is more correctly an aggregate of several (preprocessing and orbital analysis) programs accepting common data files and directed by similar user instructions.

The work performed according to the following description is representative of the actual Doppler processing, although there were inevitably some slight processing variations between data spans, and some partial explanations are tolerated for the sake of brevity. A summary of model features is given below. A more detailed description of CELEST processing is available in [5.21].

To begin the preprocessing for a data span, the observation times recorded by the tracking stations were first corrected to be consistent with one of the stable operational Navy Navigation Satellite (NAVSAT) clocks using time differences normally available every two minutes during NAVSAT passes. An additional correction was made, if

required, to bring all times close to Universal Time Coordinated (UTC). Some judgment was used to determine exactly when occasional discretionary station clock resets occurred. In a few cases where reliable time correction was impossible, time bias parameters were introduced in subsequent processing. Additional preprocessing consisted of deletion of obvious offending data, assignment of nominal accuracies to observations, and reformatting of data for CELEST orbital analysis programs.

Next, a preliminary (linearizing) orbit was fitted to the Doppler data for the span, allowing adjustment of a limited parameter set but holding fixed the gravitational coefficients and all well-known observing station coordinates. The fit was iterated to convergence giving orbits generally within 30 meters (in one case 300 meters) of truth as measured by "navigation" residuals.

Digressing briefly, "navigation" is a diagnostic single-pass least squares solution for bias parameters and displacements in two well-defined components of the station position called "navigation" residuals. The directions customarily chosen for Doppler navigation are the slant-range (from station to satellite) and along track (or parallel to the satellite velocity) directions defined at the time of closest approach (between station and satellite) for the given pass. The "navigation" residuals (which are at best the sum of orbit, station, timing, and random errors) can be interpreted as orbit errors where the other defining sources are sufficiently small. Almost by convention, the root-mean-square (RMS) of along-track "navigation" residuals for an orbit fit is quoted as a measure of orbit accuracy.

The orbit fitting procedure in the CELEST Computer Program [5.21] incorporates two iterative stages of editing, one of which also helps in the determination of accurate data weights. In each cycle of the first procedure, called point filtering, Doppler data with observation residuals (based upon a previous "navigation" solution) exceeding a constant multiple (generally 2.0 for continuous and 2.5 for sampled Doppler) of their estimated accuracies are first rejected, and the scalar

of nominal accuracies is redefined to bring the RMS of weighted residuals close to unity. The process converges here if the data and observation accuracies from the previous cycle are obtained. If the cycle limit is not exceeded, the "navigation" solution is repeated with the current specifications, and the iteration continues. Convergence is almost always obtained within 20 cycles, and the small remainder of passes is discarded.

Analyzing the residuals of observation after a "navigation" solution, rather than directly from the preliminary orbit, is preferred because of the opportunity for nearly complete elimination of orbit, station coordinate, and timing errors from the data. However, it should be noted that the procedure starts with a "navigation" solution based on Doppler data that passed an absolute tolerance test on the observation residuals from the preliminary orbit. Note also that elevation angle tolerances (generally five degrees on individual data points and 10 degrees on closest approach conditions), uniformly applied to screen out likely bad data during the above procedure, will force editing to be performed on a restricted data set which may be too small for reliable work. The procedure is not foolproof and may occasionally require manual intervention if sufficient data is to be retained.

After point filtering is completed for all passes in the data span, the second iterative procedure, called pass filtering, is performed. On each cycle of pass filtering, a new orbit is first defined using previously acceptable passes, and new "navigations" are computed. Linear functions (of the time of closest approach), representing adjustable orbit error reflected in the "navigations", are then determined in separate unweighted least-squares fits to the along-track and slant-range "navigation" residuals for acceptable passes. Finally, passes are rejected if the departure of either "navigation" residual from the predicted linear value exceeds a constant multiple (usually 2.5) of the computed RMS of all such departures for data passes previously judged acceptable. Although the process converges if the same passes are rejected here as before, the orbit is relatively insensitive to the final

editing after the most offensive passes are deleted and the program is normally instructed to treat exhaustion of the iteration count as convergence also.

Returning now to the discussion of the processing sequence, the main accomplishments of the processing at this point were the edited data, the improved orbit, and the corresponding "navigation" residuals. These residuals, together with others from a second supplementary preliminary fit (based upon a different gravitational coefficient set but the same edited observations and weights), were then used in the final manual editing of passes. Manual pass selection was performed to reverse occasional and unavoidable errors in the automatic pass rejection procedure due to large preliminary orbit errors.

Although extreme care was exercised at each stage of editing, a small amount of erroneous data undoubtedly slipped through. This is due at least in part to the pursuit of two conflicting objectives: strengthening the contribution of data sets and removing unreliable data.

After editing, the formation of normal equations from the remaining data was a relatively straightforward, although computationally expensive, task. The main steps in the procedure for the six-day data span were:

- Recomputation of the preliminary (linearizing) orbit for the given arc.

- Formation of the pass normal equations for non-gravitational parameters.

- Calculation of the variational derivatives for gravitational parameters.

- Expansion and summation of the pass equations to form arc normal equations.

The computer programs (together called TERRA) used in this work were consistent with the CELEST Computer Program and formulation [5.21], although computations were necessarily reorganized to accommodate the 1850 gravitational parameters desired in the final normal equations.

Generally, the computational procedures and models used in equation formation were identical to those employed in the above fits and the precise two-day orbit computations performed routinely at DMA and NSWC. The exceptions, including

- Reduction of the numerical integration time step from 60 to 15 seconds,

- Increasing the number of adjustable drag multipliers to two per day, and,

- General relativistic observation corrections for high eccentricity satellites,

were intended to reduce orbit errors to values below one meter in the six-day data spans used here.

The satellite equations of motion were numerically integrated using a 15-second time step with a Cowell predictor formula, neglecting the 11th difference of the force. The variational derivatives for dynamical parameters were also developed with this algorithm. In tests, the RMS errors in the six-day orbits due to these specifications were below 30 centimeters after adjustment of initial conditions. Similarly, the variational derivatives were found accurate to six digits before, and four digits after, adjustment. A partial restart was used at drag segmentation times (although not at the shadow boundary) to compensate for force discontinuities.

The satellite dynamical model used included the direct solar, lunar, and terrestrial gravitational forces, simplified lunar and solar solid-earth tidal forces, direct solar radiation pressure, and atmospheric drag (with a rotationally symmetric earth-fixed atmosphere). The Doppler satellites were modeled as spheres in the calculation of body forces. The utilized computer program permitted the determination of multipliers for the tidal and radiation forces, and (up to 20) segmented multipliers for the drag force. Studies with hypothetical drag, tidal, and radiation force model improvements suggested that residual orbit errors of those models were less than half as large as the anticipated two-meter RMS post-fit accuracy objective for medium-altitude satellites.

The observational model, based upon the above-described satellite dynamics and earth motions, was a vacuum model with propagation corrections. Modern Doppler data was treated as range difference observations. Historical sampled Doppler data was treated as range rate (frequency) observations in forming the partial derivatives, but with accurate range difference quotient observation values. Relativistic effects up to the v^2/c^2 terms (general theory) were included in Doppler computations for satellites with the more eccentric ($e>0.01$) orbits. The nearly constant effect for low eccentricity satellites was absorbed by the pass frequency-bias parameters. Frequency drift was an adjustable pass bias parameter for a few passes collected by one troublesome station. Tropospheric refraction, similar to the Hopfield bi-quartic model [5.22], was used with local (if available) or standard weather values, and scaled by an adjustable multiplier (initialized at unity with a 10 percent standard error). The station coordinate pass biases (initialized with two-meters a priori standard errors) were used to compensate for residual modeling errors.

Accuracy of the partial derivatives was generally limited to five digits as a result of neglecting v/c relativistic terms. The exception to this rule is the partial derivatives for gravitational parameters which were effectively approximated by the pass matrix

expansion method [5.21]. The accuracy of this approximation has not been adequately defined.

In the pass matrix expansion method, elements of the rows and columns of the pass normal equations corresponding to gravitational parameters are developed from the coefficients for the initial conditions exactly according to the assumption that the orbital effects of gravitational parameters can be expressed (separately for each pass) by a linear combination of the six initial condition perturbations. Equivalently, the approximation relies on there being no within-pass gravitational effects -- only effects that build slowly between passes. The linear relation is computed for a single point; the integration time step nearest to the time of closest approach for the pass. The underlying orbital approximation has been numerically tested and found accurate to a few decimeters for representative gravitational perturbations. Meanwhile, the payoff is an effective reduction to six or fewer observations per pass, improving computation costs generally by a factor of four to 40 depending on the actual number of observations.

Estimable geodetic parameters of the full Doppler normal equations for each arc consisted of coordinates of all contributing tracking sites and 1850 gravitational coefficients. The gravitational coefficient set consisted of all coefficients through degree and order 41 and the coefficients of 43 resonant (third overtone) harmonics of orders 37 through 45. Incidental parameters included satellite initial conditions, multipliers of the drag, radiation, and solid earth tidal forces, time correction parameters (if needed), and a small collection of study parameters. Bias parameters were mathematically eliminated as part of the normal equation formation procedure.

Additional work with these arc normal equations was performed before their use in the WGS 84 trial EGM solutions. First, polar motion parameters for the six-day span and a set of systematic network error parameters of the Doppler observing station network were introduced into the equations using their equivalence to station

coordinate adjustments. Then, Bayesian (a priori) observations of arc parameters (generally including force multipliers and pole parameters) were added to the normal equations. The a priori drag observations expressed the assumption that the drag coefficients were part of a steady state Markov process of indeterminate mean value. (The parameters of this process were determined experimentally for each satellite to control erratic behavior of the drag parameters in the solution.) Arc parameters, generally consisting of initial conditions, pole parameters, force multipliers, and timing parameters, were then mathematically eliminated. In the three equation sets based on 1978-era Doppler data, the earth's gravitational constant (GM) and/or the systematic network corrections, which were intended to be common parameters, had to be treated as arc parameters to relieve stress in combination solutions. (It is now believed that these free parameters absorbed residual ionospheric refraction effects.) The 13 arc-eliminated Doppler normal equations (missing HILAT Arc B) were then adjusted to a common reference parameter set, consisting of NWL 9D station coordinates and NWL 10E gravitational parameters, and combined in various groupings for inclusion in the WGS 84 EGM solution process.

The Doppler normal equations were tested individually (in tuning solutions) and in various (satellite only or general) combinations as part of an extensive validation effort. In this work, trial solutions of the equations were generally followed by orbital evaluation to quantify the degree of success obtained. The goal of the tuning experiments (two meters RMS "navigation" residuals) was met for the NAVSAT and GEOS-3 Satellites, and approached by the other modern satellites. Additional critical analysis, based upon such general criteria as small size of gravitational parameter disturbances (useful only in general combination solutions), reasonable behavior of the dynamical multipliers, and small displacements of tracking stations (from their well-surveyed NWL 9D positions) obtained in diagnostic solutions of the supporting Doppler equations after adjustment for trial EGM solution improvements, validated the remaining equations.

5.2.3 Laser

The laser measurement is a derived range obtained from the travel time to and from the satellite of a very brief pulse of monochromatic light. Propagation delays are introduced only by the troposphere and the corrections are relatively more simple to compute than those for Doppler data. Also, in contrast to the Doppler system, the laser system can improve accuracy by averaging several measurements obtained in a short time interval. Effective measurement accuracies approaching the five centimeters possible with modern systems can define two well-determined components of the satellite or tracking station positions during a pass to within a few millimeters. This accuracy level reaches far beyond requirements for gravitational potential modeling into the realm of precision relative surveying applications.

The tracking equipment, consisting of a coaxial laser emitter and photoreceptor, together with the advanced electronics necessary for precise time interval measurement, must be accurately pointed toward the target satellite. The satellite must be equipped with corner-reflector arrays pointing toward earth. Since the satellite is passive, tracking is possible throughout its orbital lifetime.

Laser tracking stations have been operated by NASA, the Smithsonian Astrophysical Observatory (SAO), and cooperating scientific agencies worldwide since the late 1960's, although the most accurate data is of recent origin. As many as 30 stations may have been operated simultaneously in observation campaigns. A database of collected data is maintained by NASA. Data is available on approximately 12 satellites, some of which were also tracked by Doppler ground-based satellite receivers.

Two satellites, Starlette [5.23] and LAGEOS [5.24], extensively tracked using the most modern laser equipment, were selected for EGM development purposes. The laser data processing and normal equations development for these satellites was performed under DoD

contract by Applied Research Laboratories/University of Texas (ARL/UT; Austin, Texas), in consultation with DMA and NSWC personnel.

The Starlette Satellite, having orbital characteristics similar to the medium-altitude Doppler geodetic satellites, was expected to provide equally valuable information for EGM determination. The laser ranging data for Starlette is much more sparse than comparable Doppler tracking would have been. Tracking density in the period examined generally averaged only two passes per day. Fortunately, the MERIT Short Campaign [5.25], August 1980 through October 1980, during which the relatively superior average daily total of eight passes was collected by the 18 station laser tracking network, was included in the period examined. In order to obtain the desired number of laser tracking passes, the entire 93-day campaign was selected for exploitation.

It should be noted that all but 125 of the approximately 750 passes in this 93-day span were collected by the nine most active stations. This 1:5 performance ratio between weak and strong stations compares unfavorably with the 1:2 performance ratio typical of the Doppler system, and does little to aid the separability of the tesseral harmonics. Balancing this, however, is the improved separability resulting from observations uniformly spanning almost a complete revolution of perigee, and the greater improvement in strength of zonal harmonic coefficient determination attainable as a result of processing long data spans.

The preprocessing of laser data, including station clock time correction to uniform atomic time, removal of some propagation effects, editing of offensive data, determination of observation accuracies, and aggregation to a reduced set of representative observations (30-second normal points) was performed using methods perfected by ARL/UT during previous exercises, and are believed to be fully compatible with WGS 84 EGM development objectives.

The entire 93-day campaign was treated as a single dynamical arc in order to extract the maximum strength for determination of gravitational coefficients. The converged linearizing orbit based upon the NASA-developed PGS-1331 EGM coefficient set [5.26] initially gave RMS laser range residuals of eight meters. This figure was reduced to 1.25 meters (a value more or less compatible with the two-meters orbit accuracy objective selected for medium altitude geodetic satellites) by adjustment of resonant gravitational coefficients up to the third overtone (order 41) and coefficients of some empirically defined periodic "drag" forces necessary to compensate suspected errors in the tidal and earth motions models that become prominent only in such very long fitting spans. Frequencies of the empirical terms were carefully checked to avoid accidental conflict with gravitational effects.

Accurate models were used in all phases of computations. The satellite equations of motion were developed by numerical integration using a 15-second time step and a predictor formula that neglected the 11th difference of the force. The variational derivatives for dynamical parameters also were integrated with this algorithm. The very complete dynamical model used in the computations included the conventional gravitational and solid-earth tidal forces, a 60-term ocean tide force similar to the MERIT standard model, solar radiation pressure, Jacchia 1971 drag model [5.27], and previously mentioned empirical forces. Earth motions included the use of the J2000.0 celestial reference [5.28], Wahr nutation [5.28] - [5.30], and geometric solid earth tides at the tracking sites. Evaluation of terrestrial gravitation involved the full transformation between the earth-fixed and celestial reference systems. The observational model included a tropospheric refraction correction based on local weather. The station coordinate pass bias parameters were available to compensate for residual model errors.

After satisfactory conclusion of the preprocessing and orbital analysis, normal equations were formed from representative laser observations at elevations of 10 or more degrees above the local horizon. Use of a two meters station coordinate pass bias a priori

standard error (as used in the Doppler medium-altitude satellite equations development) permitted the convenient replacement of variable laser observation accuracies by the uniform one meter value, thus averting potential numerical degradation of the equations.

Estimable geodetic parameters of the full laser normal equations included all 1850 gravitational coefficients chosen for the Doppler medium-altitude satellite equations together with a handful of additional resonant terms recommended by ARL/UT, and coordinates of all tracking stations. Incidental parameters included satellite initial conditions, multipliers of the radiation, drag, and solid earth tidal forces, and coefficients of the empirical periodic forces. Bias parameters were mathematically eliminated as part of the equation formation procedure.

Validation of the supporting data set and reduced normal equations (from initial orbit determination experiments) was first performed by ARL/UT prior to delivery of the full normal equations. At NSWC, tuning solutions attempted with these Starlette equations produced acceptable, but larger than expected, harmonic coefficient corrections. Orbital evaluation of these solutions, which could have further validated the equations, was not performed. One additional validation test, the annihilation of a suspected weak vector of parameter improvements (representing a shift of the common longitude reference for orbit, tracking stations, and gravitational potential) by the normal equation matrix, was successful only to four digits.

The great volume of available laser tracking for the LAGEOS Satellite [5.24] allowed a two-year-long data span to be processed into normal equations. The data span analyzed, covering 1980 and 1981 entirely, overlaps the MERIT Short Campaign [5.25] chosen for Starlette. The available LAGEOS observations represent over 4000 passes collected by 32 laser tracking stations performing at an average total of seven passes per day. LAGEOS, a satellite orbiting at a higher altitude specifically chosen to minimize terrestrial gravitational effects, was expected to

contribute mainly to the definition of the lower frequency gravitational harmonics in the WGS 84 EGM solution.

Preprocessing of LAGEOS data was conducted using techniques similar to those used in Starlette data processing, except for aggregation to three-minute (normal point) representative ranges. Subsequent orbital processing was also quite similar. As before, the entire data span was treated as a single dynamical arc. Numerical integration of the orbit and the variational derivatives was performed using a 100-second timestep more appropriate for this higher altitude satellite. Initial orbit fitting using the NASA-derived GEM-L2 coefficient set [5.31] was iterated to convergence, and yielded RMS laser range residuals of 75 centimeters, after the adjustment of empirical periodic drag parameters as required previously for Starlette. This figure is probably a little high to be consistent with the accuracy objective (of one meter RMS orbit error) set for this satellite.

Normal equations were formed from representative LAGEOS data at elevation angles above 10 degrees, using a uniform observation accuracy of 50 centimeters, and admitting one meter uncertainty in the pass station coordinate biases representing residual model error. Geodetic parameters of the normal equations included station coordinates of all contributing tracking sites and a reduced set of 312 gravitational coefficients. This reduced set includes all harmonics that are able to produce even a one centimeter RMS orbital perturbation by such large changes as 10 times the coefficient value predicted by "Kaula's rule of thumb" [5.32]. Incidental parameters were the same as previously described for Starlette equations.

Validation of the supporting data set and the reduced equations used in orbit determination experiments was conducted by ARL/UT as part of the normal equation development process. Additional gravity tuning solutions performed at NSWC confirmed the suspicion that the strength of the LAGEOS equations was concentrated mainly in 35 low-frequency gravitational terms. Again, no orbital evaluations of solutions

were performed, although laser station solutions based upon the tuned reference orbit were uniformly excellent. Equally disappointing results were obtained in the weak vector annihilation test for the LAGEOS normal equations as had been obtained in Starlette normal equation validation experiments.

5.2.4 Surface Gravity

The basic surface gravity data used in WGS 84 EGM development consisted of a 64,800-member worldwide set of 1° x 1° equi-angular mean free-air gravity anomalies. Since much of the earth's surface is either devoid of gravity measurements or has inadequate coverage, various types of data and mean gravity anomaly calculation techniques had to be used in forming this basic data set.

In January 1984, approximately 11.7 million point gravity anomalies referenced to the International Gravity Standardization Net 1971 [5.33] and the WGS 84 Ellipsoidal Gravity Formula (Chapter 4) were available for use in forming the basic 1° x 1° mean gravity anomaly data set. However, due to the irregular geographical distribution of these point gravity anomalies, only 35 percent (22,917) of the 64,800 1° x 1° mean gravity anomalies comprising the worldwide data set could be determined solely from observed gravity data. Due to the different data availability situations encountered, seven different mean gravity anomaly calculation methods were used to obtain these 22,197 1° x 1° values [5.34] - [5.36].

In oceanic areas, satellite radar altimeter data was used alone or in conjunction with observed gravity data to generate 24,400 1° x 1° mean free-air gravity anomalies. A total of 18,056 of these values was calculated using only the NAVOCEANO-developed SEASAT-1 15' x 15' "regional field" of gridded geoid heights discussed in Section 5.2.5, below. The remaining 6,344 oceanic mean gravity anomalies were computed using least squares collocation and various combinations of SEASAT-1 and GEOS-3 altimeteric geoid heights and available surface gravity data. Thus, 37.7 percent of the worldwide set of 64,800 1° x 1°

mean gravity anomalies is based wholly or in part on geoid heights deduced from satellite radar altimeter data.

From the preceding, it is apparent that 72.7 percent of the worldwide set of 1° x 1° mean free-air gravity anomalies available for use in WGS 84 EGM development was either developed solely from surface gravity data (35 percent) or from SEASAT-1 altimetric geoid heights used alone (27.9 percent) or in combination with GEOS-3 altimetric geoid heights and surface gravity data (9.8 percent). Therefore, a large percentage (72.2) of the surface gravity field used in WGS 84 EGM development is based on measured data. The remaining portion of the 1°x1° mean gravity anomaly data set (27.3 percent) was developed using either gravity/geophysical correlation techniques or a spherical harmonic expansion in which a preliminary WGS 84 EGM through n=m=41 was utilized.

Gravity/geophysical correlation techniques were used to develop 7270 1° x 1° mean free-air gravity anomalies in land areas having only sparse or no gravity measurements. An explanation of the prediction method (NOGAP) used to obtain these 1° x 1° mean gravity anomalies is given in [5.37] along with other information on gravity/geophysical correlation theory and techniques.

After using all the observational data (gravity, altimetry) available, and utilizing gravity/geophysical correlation techniques, 1° x 1° mean gravity anomalies were still needed for 10,134 surface elements to complete the worldwide data set. Mean gravity anomalies were computed for these 1° x 1° surface elements using a spherical harmonic expansion based on a preliminary WGS 84 EGM complete through n=m=41.

This worldwide field of 1° x 1° mean free-air gravity anomalies, developed as discussed above, referenced to the WGS 84 Ellipsoidal Gravity Formula, and with 1° x 1° mean terrain corrections applied where available in the northern hemisphere (Figure 5.1), is the basic set of surface gravity used in WGS 84 EGM development. Section 5.3.2 treats its use in the weighted least squares solution that generated the WGS 84 EGM

through n=m=41. Its use in generating WGS 84 EGM coefficients from n=42, m=0 through n=m=180 is discussed in Section 5.5.

5.2.5 Altimetric Geoid Heights

The geoid height data used in WGS 84 EGM development was derived from a set of approximately 4.2 million along-track point geoid heights deduced from altimeter data obtained during the abbreviated mission of SEASAT-1. Although the SEASAT-1 radar altimeter did not continuously acquire data from its activation date (3 July 1978) until instrumentation ceased to function (10 October 1978) due to a power failure, approximately 1000 revolutions of data (equivalent roughly to 70 data collection days, 14 satellite revolutions per day) were obtained while SEASAT-1 was operational [5.38].

The raw altimeter measurements were down-linked at appropriate times to one of three strategically located data acquisition stations, then collected by NASA/GSFC (Greenbelt, Maryland), and distributed to JPL (California Institute of Technology; Pasadena, California) and the Navy's Fleet Numerical Weather Center (Monterey, California), now Fleet Numerical Oceanography Center (FNOC).

At FNOC, the analog-to-digital conversion was performed and the data placed in Sensor Data Record (SDR) form. Using NSWC software, FNOC then computed atmospheric and environmental corrections utilizing the best available correction models. The data was then prefiltered and condensed from its original density of 10 data points per second to two points per second. This data, referred to in this form as Intermediate Geophysical Data Records, was then sent to NSWC for further processing. Perhaps the most significant of the environmental corrections applied to the data by NSWC, in terms of magnitude, were tide corrections computed using the Schwiderski Ocean Tide Model [5.39]. Also at NSWC, SEASAT-1 Doppler tracking data was used to develop a "tuned" EGM for use in computing precise ephemerides for the SEASAT-1 Satellite. Using the

precise ephemerides, corrected satellite-to-ocean surface altimetric range measurements, and computed satellite nadir points, point geoid heights were calculated and then smoothed using a Kalman filter. It was this along-track, two-points-per-second data that was recorded on the filtered Geophysical Data Record (GDR). Details of the NSWC SEASAT-1 Satellite radar altimetry data processing system are available in [5.40].

Magnetic tapes containing the GDRs were forwarded by NSWC to the Naval Oceanographic Office (NAVOCEANO; Bay St. Louis, Mississippi) where further data processing occurred. Substantial editing was required due to difficulties associated with the use of the weather data when computing some of the environmental corrections. After editing the data, NAVOCEANO applied a spline correction procedure to the along-track geoid heights to minimize their differences at the ground-track intersections of ascending and descending orbital arcs. This procedure was used to reduce the residual orbit error, the largest component of the altimetric geoid height error budget [5.41].

The edited and adjusted along-track data was then splined by NAVOCEANO to produce a "maximum response" 15' x 15' grid of altimetric geoid heights referenced to the WGS 72 Ellipsoid. However, visual inspection of a contoured representation of this 15' x 15' data set indicated that some residual orbit error still remained. To remove this error, NAVOCEANO returned to the edited and adjusted along-track geoid height data set and applied a 45' moving window averaging filter to the data. This operation produced a second 15' x 15' geoid height data set, referenced to the WGS 72 Ellipsoid, and referred to as the "regional field". The technique used by NAVOCEANO to correct the SEASAT-1 data for residual orbit errors is discussed in [5.42].

Magnetic tapes containing the NSWC-prepared SEASAT-1 GDRs, approximately 4.2 million along-track geoid heights, were also received by DMAAC. In addition, DMAAC obtained both "maximum response" and "regional field" 15' x 15' gridded sets of SEASAT-1 geoid heights developed by NAVOCEANO. After analyzing the three data sets, DMAAC concluded that the

NAVOCEANO-developed 15' x 15' "regional field" of gridded geoid heights best reflected the geoid height information contained in the SEASAT-1 altimeter data.

Since the NAVOCEANO-developed 15' x 15' "regional field" of gridded geoid heights was referenced to the WGS 72 Ellipsoid, the data was re-referenced to the WGS 84 Ellipsoid. From this SEASAT-1 data set, the geoid height at the center of each one degree cell was selected to form a 1° x 1° grid of geoid heights. This basic grid of SEASAT-1 derived WGS 84 Geoid Heights was then used as described in Section 5.3.3 to form a set of 3° x 3° approximately equal area mean geoid heights. The geoid height normal equation matrix used in WGS 84 EGM development was formed using this 3° x 3° gridded set of altimeter-derived mean geoid heights (Section 5.3.3).

5.2.6 NAVSTAR Global Positioning System

The NAVSTAR Global Positioning System (GPS) is an evolving DoD satellite-based radio "navigation" system with significant geodetic potential, now in transition between its demonstration and operational phases. An outgrowth of Air Force and Navy research programs of the 1960's that were consolidated in 1973, this system is scheduled to become operational in the late 1980's and will replace the Navy Navigation Satellite System in the 1990's. The new system will serve a wide user community, both military and civilian [5.43] [5.44]. In preparation for this, DMA is deeply involved in present GPS activities through its operation of tracking sites and its research in geodetic exploitation of GPS.

The GPS satellites are much like orbiting atomic clocks [5.45] [5.46], broadcasting time signals which can be read and compared against terrestrial atomic clocks at the receiving stations. The time differences from such comparisons contain the geodetically useful travel time delay, which is (except for propagation effects) proportional to the station-to-satellite range, and, inevitably, the clock calibration errors. When scaled by the velocity of light, the time differences are

called pseudo-ranges. Compensation for ionospheric propagation effects is achieved by transmitting two coherent L-band frequencies. Time difference information contained in modulating pseudo random noise (PRN) codes may be extracted by measuring the synchronizing time shift of the identical locally generated code tied to the receiving station clock. Since the carrier as well as the modulating code is controlled by the ultrastable atomic standard, excellent Doppler (or range difference) data can also be collected. The actual GPS observation is generally an amalgamation of the just described range and range difference measurements, after some processing at the tracking site.

Equipment at the tracking sites may vary, but generally consists of simple omnidirectional antennas and very sophisticated signal processing electronics operating under the control of a microcomputer. Some stations are able to track four satellites simultaneously. The collected data is generally smoothed and aggregated by microcomputer software to representative ranges at 15-minute intervals.

The main objectives of the data exploitation plan were to be served by locating, in the NSWC data library, four continuous weeks of simultaneous tracking data on five of the Phase II GPS satellites (at 63° inclination) characterized by the absence of irregularities in the atomic clock time histories, the avoidance of eclipses (a condition of entry of the GPS satellite into the earth's shadow, which introduces force modeling problems not yet solved), and participation of a large number of tracking receivers. In the span chosen, data was available as follows from eight sites: range data from four GPS monitor stations (Vandenberg AFB, California; Elmendorf AFB, Alaska; Wahiawa Naval Reservation, Oahu, Hawaii; and Andersen AFB, Guam) and integrated Doppler data from four DMA geodetic sites (Argentina, Australia, the Seychelles, and the United Kingdom). [Three of the above four GPS monitor stations have since been relocated. The five current (1987) GPS monitor stations are the Hawaii site (above), the Colorado Springs (Colorado) site, and three new sites located on the islands of Ascension, Diego Garcia, and Kwajalein.]

All selected GPS satellilte data had previously been preprocessed as part of the operational precise ephemeris computation routinely performed at NSWC. Additional preprocessing consisted of the application of clock frequency corrections, editing of outliers in accordance with operational criteria, and differencing of data initially collected as ranges to form uncorrelated range differences for input to the CELEST orbit computation program. Using the program and data, a new (linearizing) four-week orbit was determined for each GPS satellite based upon the operational earth gravitational model (WGS 72 EGM truncated to n=m=8) and station coordinates (also WGS 72 values, effectively). Finally, the data was organized into three sets by station (monitor sites, United Kingdom, and remaining DMA sites) for further processing. GPS time was used uniformly in all GPS data processing.

The orbit determination, and subsequent normal equation processing (described below), was performed using a specially modified CELEST Computer Program. In this program, the satellite orbits and variational derivatives were developed conventionally using methods identical to those used when computing operational precise ephemerides, and very similar to Doppler medium altitude orbit computation methods described earlier. Model satellites were affected by terrestrial, lunar, and solar gravitation, and the tidal, radiation (including y-axis formulation), and thrust forces. The earth model also conformed closely to the referenced medium altitude processor model except for polar motion computations as performed operationally. The adopted observational model allowed for tropospheric refraction and station pass biases, clock frequency biases treated as arc parameters, station adjustments, and orbital dynamic parameters including (in a significant departure from other CELEST processors) the gravitational coefficients. Editing of GPS data in the program was limited to exclusion of observations at elevations below 10 degrees, and data weights from the operational preprocessors were accepted without modification. No editing was performed on the resulting pass normal equations prior to formation of the arc equations.

Using the above described orbits and data, separate weekly

normal equations for each satellite and station group were formed in the modified CELEST Computer Program. Gravitational parameters consisted of 50 coefficients for harmonics up to and including the sixth degree terms. Other (largely insignificant) gravitational terms in the starting field and beyond, and the individual station coordinates, were held fixed. However, the normal equations did allow for adjustment of the earth's gravitational constant, and systematic Z-shift and scale corrections for the tracking station network. Other incidental parameters included for each satellite, clock frequency and aging parameters for each satellite and station, and, for each pass, a tropospheric refraction correction multiplier (initialized at unity with 10 percent uncertainty), and the three station biases representing residual orbit errors (initialized at two meters uncertainty). The normal equations formed in this manner were combined outside the CELEST Computer Program to give three contributing sets of equations (one for each station group) in such a way that the clock frequency parameters were effective for only one week while all other parameters were applicable to the entire four-week span.

Validation of the equations consisted of comparison of them and their solutions (for limited parameter sets) with alternate normal equations, formed using a modification of an interactive non-sequential least squares GPS data processing system used at NSWC, and their solutions. Both equations and solutions from the GPS monitor sites passed the comparison tests. However, attempts to demonstrate the superiority of the solutions to the WGS 72 field were disappointingly inconclusive. The next experiment, combining the monitor and DMA three-station equations disturbed the trial solutions so much that the DMA three-station equations (although not the data itself, except in the case of the United Kingdom) became suspect. Consequently, the GPS normal equations for WGS 84 EGM development represent only GPS monitor station tracking data for the four-week span.

5.2.7 Lumped Gravitational Coefficients

As mentioned earlier, lumped coefficients are values of certain linear combinations of gravitational coefficients responsible for

rather large orbital perturbations on particular satellites. In this Section, this same nomenclature is used to refer to the linear combinations of resonant tesseral harmonics, or of zonal harmonics, important in the analysis of medium or low altitude satellite orbits, and to combinations of low degree tesseral harmonics significant in synchronous satellite longitude accelerations.

According to investigators, principally King-Hele [5.8] - [5.13] and Wagner [5.14] - [5.19], analysis of fitted satellite orbital element histories yields estimates of the lumped gravitational coefficients which may be used as data in the estimation of the harmonic coefficients themselves. In the analysis, the investigators first corrected the fitted elements for various model errors (including solar radiation, unestimated zonal coefficients, tidal gravitation, etc.) of the fit procedure. They then computed terms of the extraction normal equations for up to five pairs of lumped coefficients using this data, and solved these equations for values of selected lumped coefficients.

This analysis was applied using elements computed over short subspans of longer tracking spans (of up to seven years duration) during which a subject satellite passed through one or more deep resonant conditions. Under those conditions, the resonant harmonics produced orbit perturbations which were both large and slow enough to be detectable in the element histories.

In the case of the tesseral resonance analysis of medium and low altitude satellites, the inclination and eccentricity histories are mainly studied. These elements define quite different (and complementary) lumped coefficients. Similarly, the zonal analysis is based upon study of nodal or eccentricity perturbations for definition of the even or odd degree coefficients. The single longitude acceleration values available from analysis of each synchronous satellite always involve the same set of low degree coefficients.

In the development of the extraction normal equations by the investigators, the orbital element histories were fitted using numerically integrated element rate variational equations for the lumped coefficients. The element histories, which may have initially been provided by a cooperating United States government agency (such as NASA or NAVSPASUR), were often recomputed by the investigators.

From an extensive literature survey conducted by NSWC, a total of 426 unique observation equations for lumped harmonics was obtained. The distribution of these equations by order observed is as follows:

Order	Equations
0	35 equations (28 odd zonal and 7 even zonal equations)
1	13 equations (synchronous satellite)
9	7 pairs of equations
10	2 "
11	7 "
12	12 "
13	9 "
14	40 "
15	73 "
16	1 "
22	2 "
26	1 "
29	11 "
30	13 "
31	10 "

with one additional unusable pair of equations for harmonics of order 45. Certain equations were deleted initially on the basis of information contained in the literature. Normal equations were then formed.

In forming these normal equations, observational values and standard errors were taken from the literature while partial derivatives for harmonic coefficients were recomputed using the best available formulations defining the influence coefficients of the linear combinations discussed by the investigators. In some cases, this produced a small improvement over the published influence coefficients. Some model error compensation was already included in the investigators' redefinition of observation weights during their own solution exercises. No additional model error compensation was performed.

The parameters of the normal equations developed from each observation generally consisted of all relevant gravitational coefficients to degree 41. The exception in the case of the synchronous satellite data of truncating above the sixth degree resulted in equations that were still effectively complete because of the especially rapid diminution of the longitude perturbations with increasing degree.

Validation of the resulting normal equations proved to be a task of greater difficulty than initially anticipated. The first exercise with the odd zonal equations, an attempt to replicate one of the published solutions, ended in uncertainty after several months of effort during which numerous discrepancies between the published observation weights and what the author's solutions must have been based upon were discovered. Use of the published, instead of the recomputed, influence coefficients was tested but judged ineffective, again because of likely differences between the published and actually used values, and possibly because of wordlength differences in the NSWC and other investigator's digital computations.

Interest then shifted to examination of observation residuals. Residuals of the lumped coefficients obtained by NSWC could not always be satisfactorily matched to published residuals using the published solutions, but agreement was generally satisfactory when using an independent EGM such as GEM 10B [5.47]. A liberal standard was still

needed to define agreement. This concluded the preliminary validation exercises.

The next step consisted of test solutions in combination with other satellite and surface data equations. Generally, only a small coefficient set, complete to degree 30, was developed in these tests (which precluded effective evaluation of some of the higher order lumped coefficient observations). Nevertheless, because the residuals from these test solutions and from GEM 10B were from five to 15 times the standard error, an additional group of 20 observations was marked for deletion. (A much larger subset of observations, with normalized residuals between four and five, was retained.)

Finally, the remaining equations were combined into two normal equations for further testing: one representing observations of the synchronous satellites and the zonal lumped coefficients, and another representing the higher order tesseral resonant observations. Test combination solutions were then performed, but, unfortunately, the EGM solutions developed with the higher order equations included degraded, rather than improved, the Doppler residuals for the HILAT Satellite. (Doppler data from the HILAT Satellite had been excluded from the test EGM solutions.) Therefore, as a matter of safety, and because of relatively lesser familiarity with it, the resonant tesseral equation set was dropped to resolve the apparent conflict, leaving only the low order equations to assist in the development of the final WGS 84 EGM.

5.3 Determination of Gravitational Coefficients ($\bar{C}_{n,m}, \bar{S}_{n,m}$)

The WGS 84 EGM consists of 32755 coefficients, and is complete through degree (n) and order (m) 180. These coefficients were determined in two separate solutions, or processes, with a worldwide 1° x 1° mean free-air gravity anomaly data set being common to both solutions. First, normal equations were formed for the various types of data discussed in Section 5.2 and used in a weighted least squares solution to obtain gravitational coefficients through n=m=41, the first part or low degree

portion of the WGS 84 EGM. The formation and use of these normal equations to obtain the first portion of the WGS 84 EGM, gravitational coefficients through n=m=41, are discussed in this Section. Evaluation of these gravitational coefficients (through n=m=41) is discussed in Section 5.4. The determination of the second portion of the WGS 84 EGM, gravitational coefficients from n=42, m=0 through n=m=180, is treated briefly in Section 5.5.

5.3.1 Parameters of the Normal Equations

The principal geodetic parameters of the normal equations, as mentioned before, are the spherical harmonic coefficients of the gravitational potential through degree and order 41 (numbering 1761). The decision to use spherical harmonic coefficients (in preference to an alternate parameterization) was motivated by historical precedent, the knowledge that the harmonics were the solution to the underlying potential problem, and the expectation of equivalent efficiency of representation and model evaluation in the wide variety of applications considered by the WGS 84 Development Committee.

Additional parameters in the solution were potential biases introduced to absorb any difference between the gravity potential of the earth (or geoid) and the theoretical gravity potential of the WGS 84 Ellipsoid figuring in the gravity anomaly and altimetric geoid height data processing. Other parameters included the systematic error parameters of the Doppler Coordinate System (NSWC 9Z-2) previously discussed and other less significant parameters, such as the coordinates of satellite tracking stations, and various other bias and model error compensation parameters introduced in the data processing.

5.3.2 Surface Gravity Normal Equations

The 1°x1° mean free-air gravity anomaly file (Section 5.2.4) was merged into 4,584 3°x3° approximately equal-area mean free-air gravity anomalies. These gravity anomalies were defined in terms of 3°

latitude bands subdivided into whole degree longitude increments. The standard errors used for weighting, as well as the values for the mean gravity anomalies, were developed directly from the corresponding values and standard errors of the 1°x1° mean free-air gravity anomalies in the basic 1° x 1° file. The observation equations for forming the mean gravity anomaly normal matrix were of the form:

$$\overline{\Delta g} = \frac{\int_s \Delta g \, ds}{\int_s ds} \tag{5-1}$$

or, in terms of geocentric latitude (ϕ') and longitude (λ):

$$\overline{\Delta g} = \frac{\int_{\phi_1'}^{\phi_2'} \int_{\lambda_1}^{\lambda_2} \Delta g \, r^2 \cos\phi' \, d\phi' \, d\lambda}{\int_{\phi_1'}^{\phi_2'} \int_{\lambda_1}^{\lambda_2} r^2 \cos\phi' \, d\phi' \, d\lambda} \tag{5-2}$$

where, for a surface element:

$$ds = r^2 \cos \phi' \, d\phi' \, d\lambda \tag{5-3}$$

ϕ_1', ϕ_2' = geocentric latitude boundaries

λ_1, λ_2 = longitude boundaries

$\overline{\Delta g}$ = mean gravity anomaly .

The mean gravity anomaly matrix was formed from the observation equations and mean gravity anomaly data set as described above. The constants, such as the earth's gravitational constant (GM), semimajor axis of the ellipsoid (a), the ellipsoidal flattening (f), the earth's rotation rate (ω), and zonal harmonics, were either WGS 84

defining parameters or derived from them. The weighting scheme used for each observational element, i, was

$$W_i = 1/(\sigma_i^2 + \sigma_m^2) \qquad (5\text{-}4)$$

where:

W_i = Weight of the ith observation

σ_i = Standard error for the ith observation (developed from the 1°x1° standard errors)

σ_m = Model error used for each "observation". This represents the limitation of a truncated model (n=m=41) to represent 3°x3° equal-area mean gravity anomalies. (The value used for σ_m was ±10 milligals.)

5.3.3 Altimetric Geoid Height Normal Equations

The 1°x1° geoid height data file generated from SEASAT-1 observations (Section 5.2.5) was merged into 2,918 3°x3° approximately equal-area mean geoid heights. As with the mean gravity anomaly data set, the geoid height means were defined in terms of 3° latitude bands subdivided into whole degree longitude increments. The available data covered non-land areas between 60° south and 69° north latitudes. In ocean/land interface areas, at least two-thirds of a 3°x3° surface element had to have altimetry coverage before that element could be selected for the 3°x3° equal area mean geoid height data file. For these boundary elements, the standard errors were significantly larger than for surface elements in the open ocean.

The observation equations for forming the mean geoid height normal equation matrix were of the form:

$$\bar{N} = \frac{\int_S N ds}{\int_S ds} \tag{5-5}$$

or, in terms of geocentric latitude (ϕ') and longitude (λ) :

$$\bar{N} = \frac{\int_{\phi_1'}^{\phi_2'} \int_{\lambda_1}^{\lambda_2} N r^2 \cos\phi' \, d\phi' \, d\lambda}{\int_{\phi_1'}^{\phi_2'} \int_{\lambda_1}^{\lambda_2} r^2 \cos\phi' \, d\phi' \, d\lambda} \tag{5-6}$$

where, for a surface element:

$$ds = r^2 \cos \phi' \, d\phi' \, d\lambda \tag{5-7}$$

ϕ_1', ϕ_2' = geocentric latitude boundaries

λ_1, λ_2 = longitude boundaries

$\bar{N}$ = mean geoid height .

The mean geoid height normal equation matrix was formed from the observation equations and geoid height data set described above. As in the mean gravity anomaly normal equations, WGS 84 parameters were used in

forming the observation and normal equations. The weighting scheme used for each observation element, i, was

$$W_i = 1/(\sigma_i^2 + \sigma_m^2) \qquad (5\text{-}8)$$

where:

W_i = Weight of the ith observation

σ_i = Standard error for the ith observation(developed from the 1° x 1° standard errors)

σ_m = Model error used for each "observation". This represents the limitation of a truncated model (n=m=41) to represent 3°x3° equal-area mean geoid heights. (The value used for σ_m was ±1 meter.)

5.3.4 Results

The WGS 84 EGM through degree (n) and order (m) 41 represents the solution of a normal equation matrix formed by combining normal equation matrices developed from 3°x3° equal-area mean free-air gravity anomaly "observations", 3°x3° equal-area mean geoid height "observations" in ocean areas covered by SEASAT-1 radar altimetry, and numerous normal equation matrices formed from Doppler or laser satellite tracking data. Normal equation matrices were also included for NAVSTAR GPS data and "lumped coefficients" data, respectively. Since a combined mean gravity anomaly and mean geoid height normal equation matrix can produce a relatively good n=m=41 EGM not obtainable from other data, an EGM solution based on this combined matrix became the initial preliminary WGS 84 EGM. The individual satellite normal equation matrices were then added to this combined matrix one or two at a time to obtain all

subsequent preliminary WGS 84 EGMs. This orderly approach permits the identification of a potential problem arising from a particular data set. The solution coefficients for the gravity anomaly/geoid height combined matrix as well as the many intermediate solutions, up to and including the final WGS 84 EGM, were tested and evaluated. Since an attempt was made to account for the model error during the formation of the individual normal equation matrices, all of the normal equation matrices were added together using a scalar quantity of one.

5.4 Evaluation of the WGS 84 EGM (Through n=m=41)

5.4.1 General

Currently, an absolute standard is not available for evaluating an earth gravitational model external to the earth's surface. Therefore, a fully satisfactory evaluation of the WGS 84 EGM cannot be made. However, a number of calculations and comparisons can be made that provide information on the model's overall quality and relative merits. These include, for example, use of the EGM:

- To calculate gravity anomaly degree variances which are then compared with similar values from other known models.

- To calculate mean gravity anomalies and mean geoid heights for comparison with similar data.

- To perform various satellite orbit analyses.

This Section discusses such comparisons and analyses performed with the n=m=41 portion of the WGS 84 EGM.

5.4.2 Degree Variances

As a first step in model evaluation, gravity anomaly

degree variances were computed for each intermediate model as it was developed. This is a relatively simple computation that serves as a problem indicator when larger than expected magnitudes occur for the normalized coefficients ($\bar{C}_{n,m}, \bar{S}_{n,m}$). Such a problem may indicate a high correlation between gravitational model coefficients, inadequate information content in the data sets, or an error in forming an intermediate normal equation matrix.

Gravity anomaly degree variances (c_n) are computed by the equation [5.48]:

$$c_n = \bar{\gamma}^2 (n-1)^2 \sum_{m=o}^{n} (\bar{C}^2_{n,m} + \bar{S}^2_{n,m}) \qquad (5\text{-}9)$$

where $\bar{\gamma}$ is the average value of theoretical (normal) gravity (Chapter 4). Degree variances of the gravitational potential (σ_n^2) are sometimes computed instead of gravity anomaly degree variances. The relevant equation is [5.32]:

$$\sigma_n^2 = \sum_{m=o}^{n} (\bar{C}^2_{n,m} + \bar{S}^2_{n,m}) \qquad (5\text{-}10)$$

The following expression, with linkage to Equation (5-10), and often referred to as "Kaula's Rule-of-Thumb" [5-32], is sometimes used to provide an estimate of the magnitudes of the $\bar{C}_{n,m}$ and $\bar{S}_{n,m}$ coefficients for a given degree, n:

$$\sigma_n (\bar{C}_{n,m}, \bar{S}_{n,m}) = \frac{\sigma_n}{(2n+1)^{1/2}} \simeq \pm \frac{10^{-5}}{n^2} \qquad (5\text{-}11)$$

For purposes of comparison, gravity anomaly degree variances were computed for various models available in the literature. The OSU 81 EGM [5.3] and GEM 10C EGM [5.47], in particular, were used extensively in WGS 84 EGM comparison and evaluation studies.

Gravity anomaly degree variances through n=m=41 are shown in Table 5.2 for the GEM 10C and OSU 81 EGMs, and the least squares portion of the WGS 84 Earth Gravitational Model, WGS 84 (n=m=41). In

addition, solutions for the WGS 84 (n=m=41) normal equation matrix were made for a 36th degree and order model (WGS 84 n=m=36), for a 30th degree and order model (WGS 84 n=m=30), and for a 24th degree and order model (WGS 84 n=m=24). These lower degree and order EGM solutions were obtained in order to determine if the aliasing of higher degree and order gravitational effects into lower degree and order coefficients had occurred in the WGS 84 EGM (n=m=41). From the gravity anomaly degree variances (Table 5.2), it is apparent that the WGS 84 EGM (n=m=41) is not affected by aliasing since significant aliasing is not present in the lower degree and order experimental models.

5.4.3 Mean Gravity Anomaly Comparisons

One method of evaluating an earth gravitational model is to compute the RMS difference between mean gravity anomalies developed from spherical harmonics using an EGM ($\overline{\Delta g}_h$) and mean gravity anomalies developed from terrestrial data (Δg_t) [5.49]. The terrestrial mean gravity anomalies used for the comparison were developed from observed data only and categorized in terms of percentage observed. For example, in the case of 3°x3° equal-area mean gravity anomalies, 100 percent observed indicates that a value exists for each 1°x1° equal-area mean gravity anomaly contained in the 3°x3° equal-area mean gravity anomalies. In the case of 67 percent observed, at least six out of the nine 1°x1° equal-area mean gravity anomalies required to develop a 3°x3° equal-area mean gravity anomaly were available. Similar logic was used for categorizing observed percentages for the 5°x5° and 10°x10° equal-area mean gravity anomaly comparisons.

The WGS 84 EGM (n=m=41) coefficients were used in a spherical harmonic expansion to compute 3°x3°, 5°x5°, and 10°x10° equal-area mean gravity anomalies. These computed mean gravity anomalies were then compared to their counterparts developed from observed terrestrial data. (Equal-area mean gravity anomalies developed from altimetry were considered as "observed" as well as those developed from surface gravity observations.) The RMS differences resulting from the comparisons, are

shown in Tables 5.3, 5.4, and 5.5. Comparisons were also made using different truncation levels (n=m=41, n=m=36, n=m=30, n=m=24) in order to obtain an indication of the informational content of harmonic coefficients above various degree and order truncation levels. In all instances, except for the smoother longer wavelength 10°x10° equal-area mean gravity anomalies, better agreement between the computed and "observed" mean gravity anomalies was obtained for the untruncated n=m=41 model. Better agreement was also obtained for the 100 percent observed data sets which was the intended result of the higher weights used for the better observed mean gravity anomalies.

A series of experimental earth gravitational models were developed to assist in the analysis of the WGS 84 EGM. As mentioned earlier, solutions were made for n=m=36, n=m=30, and n=m=24 experimental models. The analysis shows that a truncation of the WGS 84 EGM is slightly superior to models developed as independent solutions of the WGS 84 normal equation matrix (Tables 5.3 - 5.5). It also indicates that the aliasing of higher degree and order wavelength information into lower degree and order coefficients is practically nonexistent for the WGS 84 data set.

An experimental EGM (n=m=41) was also developed and tested in which the geoid height data was eliminated from the WGS 84 normal equation matrix. This model was developed to investigate the possibility of a conflict between the mean gravity anomaly and mean geoid height information and also because some of the basic mean gravity anomaly data (1°x1° geographic area means) was developed from altimetry. This duplication could conceivably overweight the surface information in these areas. The tests showed better agreement between the WGS 84 EGM and the "observed" terrestrial mean gravity anomalies when the geoid information was included (Tables 5.3 - 5.5). This leads to the conclusion that the mean geoid height information complements the mean gravity anomaly data. However, the same result might have been obtained by increasing the relative scaling of the mean gravity anomaly matrix with respect to the combined satellite normal equation matrix. (The relative scaling used in combining all of the normal equation matrices was unity.)

A WGS 84 mean gravity anomaly/mean geoid height combination solution was developed to investigate the effect of the satellite information on terrestrial data (mean geoid heights and mean gravity anomalies) comparisons. As expected, the comparison shows that this mean gravity anomaly/mean geoid height model reproduces the terrestrial mean gravity anomalies better than the WGS 84 EGM, which included all of the additional satellite information. The improvement is on the order of 0.5 milligal or less (Table 5.3). This is a small price to pay for the very significant improvement for orbit reduction applications when the satellite information is included as in the WGS 84 EGM development.

5.4.4 Mean Geoid Height Comparisons

The principles involved in gravitational model evaluation by comparing mean gravity anomalies can be extended to include mean geoid height comparisons. However, such a comparison is limited to oceanic mean geoid heights determined from satellite radar altimetry. The basic geoid height data set consists of equi-angular 1°x1° mean geoid heights (Section 5.2.5) which are developed into 3°x3°, 5°x5°, and 10°x10° equal-area mean geoid heights in much the same way as the equal-area mean gravity anomalies were developed. The 100-percent observed data lies entirely within the ocean and within the latitude boundaries of SEASAT-1 coverage. Less-than-100-percent observed implies ocean/land interface areas in which a minimum of percentage observed is ocean surface. The smaller the percentage observed, the less accurate the equal-area mean geoid heights. The EGMs evaluated are identical to those described in the preceding mean gravity anomaly comparisons. Tables 5.6, 5.7, and 5.8 list the results of these tests. Results from the 100-percent observed category (Table 5.6), covering slightly more than one-half of the earth's surface, indicate that the n=m=41 WGS 84 EGM reproduces "observed" 3°x3° equal-area mean geoid heights at the ±1.05 meter level. This is consistent with the assumed accuracy (one sigma) of the observational data, including the model error previously discussed. Even better agreement is obtained for the 5°x5° and 10°x10° comparisons (Tables 5.7 and 5.8, respectively). This is as expected since these equal-area means are smoother than the 3°x3° equal-area

means, and because the n=m=41 model is to some extent overdetermined for these larger area mean geoid heights. As in the case of the mean gravity anomaly comparisons, results from a truncation to lower degree and order of the WGS 84 EGM (n=m=41) are approximately equivalent to those obtained using a corresponding "solved for" lower degree and order EGM (Tables 5.2 - 5.8). (The latter EGMs are denoted by WGS 84* in the tables.) The RMS geoid height differences in the 100-percent observed columns of Tables 5.6 and 5.7 show an 0.5 meter increase (approximately) for mean geoid heights generated using the n=m=41 EGM from the least squares solution in which the mean geoid height normal equation matrix was excluded as opposed to mean geoid heights generated using the official WGS 84 n=m=41 EGM (first row of each table). (In Table 5.6, 1.59 exceeds 1.05 by 0.54, and in Table 5.7, 1.38 exceeds 0.89 by 0.49.) A smaller increase occurred for analogous comparisons involving 10°x10° equal area mean geoid heights. (In the 100-percent observed column of Table 5.8, 1.16 exceeds 0.84 by 0.32.) The RMS geoid height differences in the 100-percent observed columns of Tables 5.6 - 5.8 show only a 10 centimeter improvement (approximately) for mean geoid heights generated using the n=m=41 EGM from the least squares solution in which only the mean gravity anomaly and mean geoid height normal equation matrices were used as opposed to mean geoid heights generated using the official WGS 84 n=m=41 EGM. (In Table 5.6, 0.96 is 0.09 less than 1.05 and in Table 5.7, 0.80 is 0.09 less than 0.89. In Table 5.8, 0.74 is 0.10 less than 0.84.) This indicates an excellent balance between the surface and satellite data used to form the WGS 84 EGM normal equation matrix.

5.4.5 Coefficient Errors

The preceding mean gravity anomaly and mean geoid height comparisons, plus the satellite orbital analysis results contained in [5.50], tend to be EGM validation tests rather than coefficient accuracy tests. This is because these comparisons are not independent tests, since the data being used to validate the EGM is for the most part very similar to the information used in its development. Such tests, however, do show that the data sets were integrated correctly into the EGM development process in a relative sense, because the finished model is shown to be equally good for orbit reduction applications and mean gravity anomaly or geoid height

computations. It is thus an excellent general purpose EGM that performs as well as special purpose models. That is, the WGS 84 EGM, based on a combination of surface and satellite data, performs as well in orbit reduction applications as a "tuned" EGM based solely on satellite data.

The question of EGM accuracy is distinctly different from that of validation in that accuracy implies how well the EGM represents the "true" gravitational field of the earth. However, the total error, as represented by the difference between the unknown "true" field and that represented by a set of gravitational coefficients to a given degree and order, must be broken down into two parts - the error of commission and the error of omission. The accuracy discussion here is limited to the error of commission or the error resulting from gravitational model coefficient errors up through n=m=41. (Again, this represents the degree and order of the least squares portion of the WGS 84 EGM.)

The approach often taken to define gravitational model coefficient accuracy for a model developed using least squares techniques is to use the standard errors, developed from the inverse normal matrix. This is a satisfactory approach if the standard errors (of the observations) used for weighting the various observations are correct, and if the model error assumed for each data set (as previously discussed) is correct. Sensitivity of the "observed" quantities - mean gravity anomalies, mean geoid heights, and satellite orbital perturbations - to the gravitational model coefficients or any other factor that affects the magnitude of the elements of the normal matrix, and in turn the magnitude of the elements of the inverse normal matrix, can lead to standard errors for the coefficients that are either too large or too small to represent the true error of commission. The determination of a scaling factor, if required, is a complex task. Other complications also arise when (if) the off-diagonal elements of the correlation matrix are too large.

Computations were made to determine the sensitivity of individual EGM coefficients to the 10 satellites providing data for the WGS 84 EGM solution. The coefficient sensitivity was expressed in terms of satellite orbital perturbations. (Lumped gravitational coefficient data did not

figure in the computational analysis.) Table 5.9 shows for each WGS 84 EGM coefficient through n=m=41 the number of satellites (from 1 to 10) for which the coefficients produce orbital perturbations equal to or greater than 0.5 meter. Due to lack of sensitivity, those coefficients, for which an entry is lacking in Table 5.9, were determined primarily from the mean gravity anomaly and/or mean geoid height data used in the solution rather than from satellite data. This table also indicates the strength of the satellite information for determining the lower degree and order coefficients and the first and second order resonant coefficients. One might also expect that a relatively high correlation would exist between the satellite sensitive coefficients, due to the similarity of the period of the orbital perturbations of a satellite orbit for a given order coefficient. This "lumped" effect can be reduced and the correlation minimized by including data from a large variety of satellites with different orbital characteristics. The correlation matrix for the WGS 84 EGM coefficients (through n=m=41) is summarized in Table 5.10. Correlation coefficients smaller than 0.5 are not included in the table.

Table 5.10 reflects the influence of the satellite data in that the coefficients with the highest correlation coefficients are also the ones highly sensitive to orbital perturbations. This high correlation may complicate the problem of determining "true" EGM coefficient error, but does not necessarily present a problem in terms of some applications - particularly orbit determination - because the total or "lumped" effect is of primary importance rather than the individual effect of a particular gravitational coefficient. For this reason, the formal coefficient errors (1σ) developed from the inverse normal equation matrix would still be adequate for orbit analysis regarding the expected error due to uncertainties in the WGS 84 EGM.

The WGS 84 EGM coefficient errors are discussed in [5.50] for coefficients through n=m=41. Error values are not available for coefficients above n=m=41.

5.5 Independent Spherical Harmonic EGM Determination (n=m=180)

A spherical harmonic expansion of the earth's gravitational field

through n=m=41, although suitable for satellite orbit calculation and prediction purposes, is incapable of modeling the detailed gravitational field variations important in earth and near-earth gravimetric applications. To account for these variations in support of such applications, small-size ($\leq$5'x5') mean gravity anomalies are used with surface integral formulation or along with similar size mean geoid heights to generate point mass sets which are subsequently used in relatively simple calculation formulas. For applications requiring high accuracy, such approaches (and data sets) are more appropriate than a spherical harmonic expansion (with attendant EGM coefficients) for accounting for the effect of detailed variations in the earth's gravitational field. Unfortunately, the coverage and accuracy of small-size ($\leq$5'x5') mean gravity anomalies is limited. However, gravity coverage and gravity station density continues to improve, and has improved considerably in oceanic areas since January 1984 due to the continuation of ocean gravity surveys by NAVOCEANO and the availability of GEOSAT Satellite radar altimeter data. As a result, and until quality spherical harmonic expansions (EGMs) greater than n=m=180 are in use, development of an EGM to n=m=180, and its use for earth and near-earth gravimetric applications over much of the earth has considerable practical value.

The WGS 84 EGM from n=m=41 through n=m=180 was developed from a spherical harmonic analysis of a worldwide 1°x1° residual mean free-air gravity anomaly field. This worldwide residual field was formed by subtracting from the individual 1°x1° mean free-air gravity anomalies comprising the data set discussed in Section 5.2.4 the contribution of the WGS 84 EGM coefficients through n=m=41 previously determined from various types of data utilizing a weighted least squares solution (Section 5.3). In performing the spherical harmonic analysis for the high degree and order WGS 84 EGM coefficients, the 1°x1° residual mean free-air gravity anomalies were assumed to have equal accuracy; i.e., no weighting was applied to the data used in the solution.

The high degree and order WGS 84 EGM coefficients (n=42, m=0 through n=m=180) resulting from the spherical harmonic analysis of the worldwide 1°x1° residual field were combined with the low degree and order portion (n=2, m=0 through n=m=41) of the WGS 84 EGM obtained from the

weighted least squares solution to form the WGS 84 EGM (n=m=180). Additional information on the n=m=180 WGS 84 EGM and its availability is provided in Section 5.6.

5.6 Summary/Availability of WGS 84 EGM Coefficients

The official WGS 84 EGM is complete through n=m=180, consisting of 32755 gravitational coefficients. While such a model is needed for certain applications on the earth's surface and in near-earth space, it is not required and is impractical to apply for many applications. However, the WGS 84 EGM through n=m=180 is to be used when calculating WGS 84 Geoid Heights, gravity disturbance components (or deflection of the vertical components), or 1°x1° mean gravity anomalies via a spherical harmonic expansion. Expansions to this degree and order (n=m=180), and higher, are needed to accurately model variations in the earth's gravitational field on or near the earth's surface.

The WGS 84 EGM through n=m=41 is more appropriate for satellite orbit calculation and prediction purposes. The use of higher degree and order models for such applications is not recommended at this time. However, for each satellite orbit class, DMA and other DoD users will need to conduct orbital analyses to ascertain the EGM truncation level that is "best" for the satellite involved, document results, coordinate recommendations (as necessary), and then use this particular truncated EGM for the satellite being analyzed.

As stated previously, the WGS 84 EGM coefficients through n=m=41 were obtained from a weighted least squares solution of a normal equation matrix developed by combining individual normal equation matrices formed from Doppler satellite tracking data, satellite laser ranging data, surface gravity data, oceanic geoid heights deduced from satellite radar altimeter data, NAVSTAR GPS data, and lumped coefficients. The effect (contribution) of these coefficients was removed from the worldwide 1°x1° mean free-air gravity anomaly field that contributed to their determination, leaving a worldwide residual 1°x1° mean free-air gravity anomaly field. The WGS 84 EGM coefficients from n=42, m=0 through n=m=180 were then determined

independently via harmonic analysis using the residual field. The coefficients through n=m=41 from the weighted least squares solution and the coefficients above n=m=41 from the independent harmonic analysis comprise the n=m=180 WGS 84 EGM.

The WGS 84 EGM through n=m=180 is available on magnetic tape in normalized form. The WGS 84 EGM through n=m=41 is available on a separate magnetic tape in both normalized and conventional form. The WGS 84 EGM coefficients through n=m=18 are available in [5.51] and [5.52] in normalized form. Requesters with a need for a portion of the WGS 84 EGM above n=m=18 should forward their requirements to the address provided in the PREFACE.

As stated previously, error values are not available for all of the WGS 84 EGM coefficients. However, an error covariance matrix is available for those coefficients through n=m=41 determined from the weighted least squares solution. Requesters having a need for this error data should forward their correspondence to the address provided in the PREFACE. Gravity anomaly degree variances are available in [5.51] and [5.52] for the WGS 84 EGM (n=m=180).

SECURITY CLASSIFICATION: The WGS 84 EGM (n=m=180) is classified CONFIDENTIAL, with the exception that the portion of the model through n=m=18 is UNCLASSIFIED. However, the UNCLASSIFIED portion of the EGM should be considered for use only in two instances:

- When a situation or application requires the use of an UNCLASSIFIED EGM, thereby precluding use of the CLASSIFIED WGS 84 EGM.

- When satellite orbital analyses reveal that the WGS 84 EGM truncated at n=m=18 (or lower) provides results of acceptable accuracy.

Error values for the WGS 84 EGM coefficients above n=m=18 are classified CONFIDENTIAL.

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Table 5.1

Satellites Providing Doppler Data
For WGS 84 EGM Development

Satellite Name	Orbit Characteristics		
	Inclination	Perigee Height	Semimajor Axis
DB-14	63°	971 km	7487 km
HILAT	82	768	7179
NNS 68	90	949	7457
SEASAT-1	108	778	7159
GEOS-3	115	818	7216
* BEACON-C	41	941	7503
* GEOS-1	59	1113	8072

* Historical Doppler Satellites

Table 5.2

Gravity Anomaly Degree Variances (c_n)
(Units = Milligals^2)

Degree	Earth Gravitational Models					
	OSU 81	GEM 10C	WGS 84 (n=m=41)	WGS 84* (n=m=36)	WGS 84* (n=m=30)	WGS 84* (n=m=24)
2	7.6	7.6	7.6	7.6	7.6	7.6
3	33.8	33.9	33.9	33.9	34.0	34.0
4	19.3	19.3	19.2	19.2	19.2	19.2
5	20.7	21.0	20.9	21.0	20.9	20.9
6	19.3	19.3	19.4	19.4	19.4	19.4
7	20.1	19.4	19.3	19.4	19.5	19.5
8	11.0	11.4	10.9	10.9	10.9	11.1
9	11.1	11.3	11.5	11.5	11.4	11.4
10	9.8	9.6	9.7	9.7	9.7	9.5
11	6.9	6.1	6.4	6.3	6.3	6.1
12	3.2	2.9	2.6	2.6	2.5	2.4
13	7.2	7.6	7.4	7.4	7.3	7.3
14	3.0	3.4	3.2	3.2	3.2	3.4
15	3.0	3.6	3.4	3.4	3.4	3.3
16	4.8	4.2	3.9	3.9	4.1	4.3
17	3.5	3.2	3.6	3.6	3.6	4.3
18	3.3	4.3	3.6	3.6	3.6	3.7
19	3.0	4.3	3.3	3.4	3.5	3.6
20	1.9	3.1	3.1	3.2	3.2	3.5
21	2.4	3.5	3.2	3.3	3.3	3.5
22	3.7	4.2	3.5	3.6	3.9	4.3
23	2.4	2.1	2.7	2.8	3.1	2.8
24	2.1	2.6	2.6	2.6	2.8	2.2
25	2.6	3.0	2.9	2.9	2.9	
26	1.7	2.4	2.4	2.6	2.9	
27	1.6	1.8	1.9	2.0	2.5	
28	2.3	3.2	2.4	2.6	3.4	
29	1.9	2.3	2.4	2.6	2.7	
30	2.5	3.9	2.8	3.0	3.2	
31	1.6	1.8	2.9	2.9		
32	1.6	1.6	4.1	4.1		
33	2.0	2.9	3.4	3.9		
34	2.8	2.8	5.0	4.6		
35	2.3	3.3	4.4	4.4		
36	1.7	2.0	3.6	2.8		
37	3.2	2.8	3.4			
38	2.6	1.7	2.8			
39	3.4	2.4	3.5			
40	2.8	2.3	3.6			
41	3.1	2.5	2.8			

* Experimental WGS 84 EGMs; do not use for official DoD applications

Table 5.3

Comparison of 3°x3° Equal-Area Mean Gravity Anomalies Computed From Earth Gravitational Models With Those Derived From Terrestrial Data (Units = Milligals)

Earth Gravitational Model	Truncation Level (Degree)	RMS Differences		
		33% Obs n = 4007	67% Obs n = 3679	100% Obs n = 3190
WGS 84	41	± 9.31	± 8.44	± 7.53
	36	9.78	8.99	8.15
	30	10.44	9.73	8.93
	24	10.90	10.26	9.47
WGS 84 Experimental Earth Gravitational Models				
WGS 84*	36*	9.85	9.03	8.15
	30*	10.56	9.84	8.95
	24*	11.03	10.37	9.50
WGS 84 Less Geoid Height Matrix Data	41	9.32	8.52	7.79
	36	9.82	9.08	8.33
	30	10.51	9.84	9.10
	24	10.97	10.36	9.60
	18	11.50	10.91	10.09
WGS 84 Gravity Anomaly and Geoid Height Matrices Only	41	8.80	8.02	7.34
	36	9.35	8.62	7.97
	30	10.12	9.45	8.79
	24	10.67	10.07	9.38
	18	11.33	10.73	9.98

n is the number of squares included in the sample out of a possible 4584 values (worldwide coverage).

* Least squares solution made to the degree and order indicated using the WGS 84 data set (normal equation matrix).

Table 5.4

Comparison of 5°x5° Equal-Area Mean Gravity Anomalies Computed From Earth Gravitational Models With Those Derived From Terrestrial Data (Units = Milligals)

Earth Gravitational Model	Truncation Level (Degree)	RMS Differences		
		40% Obs n = 1421	80% Obs n = 1238	100% Obs n = 1036
WGS 84	41	± 5.72	± 4.23	± 3.66
	36	6.11	4.66	4.14
	30	6.67	5.24	4.63
	24	7.27	5.89	5.20
WGS 84 Experimental Earth Gravitational Models				
WGS 84*	36*	6.15	4.65	4.08
	30*	6.74	5.33	4.66
	24*	7.41	5.98	5.21
WGS 84 Less Geoid Height Matrix Data	41	5.83	4.54	4.13
	36	6.20	4.89	4.47
	30	6.76	5.46	4.94
	24	7.34	6.07	5.44
	18	8.04	6.72	6.09
WGS 84 Gravity Anomaly and Geoid Height Matrices Only	41	5.11	3.98	3.52
	36	5.53	4.41	3.94
	30	6.21	5.03	4.48
	24	6.92	5.77	5.13
	18	7.77	6.61	5.98

n is the number of squares included in the sample out of a possible 1654 values (worldwide coverage).

* Least squares solutions made to the degree and order indicated using the WGS 84 data set (normal equation matrix).

Table 5.5

Comparison of 10°x10° Equal-Area Mean Gravity Anomalies Computed From Earth Gravitational Models With Those Derived From Terrestrial Data (Units = Milligals)

Earth Gravitational Model	Truncation Level (Degree)	RMS Differences		
		40% Obs n = 364	80% Obs n = 299	100% Obs n = 208
WGS 84	41	± 2.94	± 1.95	± 1.52
	36	2.93	1.95	1.51
	30	3.06	2.02	1.60
	24	3.29	2.11	1.79
WGS 84 Experimental Earth Gravitational Models				
WGS 84*	36*	2.95	1.96	1.52
	30*	3.18	2.12	1.67
	24*	3.45	2.21	1.82
WGS 84 Less Geoid Height Matrix Data	41	3.25	2.26	1.98
	36	3.24	2.26	1.98
	30	3.35	2.34	2.11
	24	3.57	2.44	2.26
	18	3.95	2.82	2.54
WGS 84 Gravity Anomaly and Geoid Height Matrices Only	41	2.71	1.77	1.42
	36	2.71	1.77	1.40
	30	2.81	1.82	1.46
	24	3.01	1.94	1.71
	18	3.70	2.62	2.32

n is the number of squares included in the sample out of a possible 416 values (worldwide coverage).

* Least squares solutions made to the degree and order indicated using the WGS 84 data set (normal equation matrix).

Table 5.6

Comparison of 3°x3° Equal-Area Mean Geoid Heights
Computed From Earth Gravitational Models With Those
Derived From SEASAT-1 Geoid Height Data
(Units = Meters)

Earth Gravitational Model	Truncation Level (Degree)	RMS Differences		
		33% Obs n = 3101	67% Obs n = 2918	100% Obs n = 2672
WGS 84	41	± 1.55	± 1.28	± 1.05
	36	1.64	1.38	1.16
	30	1.76	1.53	1.30
	24	1.91	1.70	1.46
WGS 84 Experimental Earth Gravitational Models				
WGS 84*	36*	1.62	1.35	1.12
	30*	1.74	1.49	1.25
	24*	1.88	1.67	1.42
WGS 84 Less Geoid Height Matrix Data	41	1.99	1.79	1.59
	36	2.03	1.84	1.63
	30	2.12	1.93	1.71
	24	2.22	2.04	1.82
	18	2.40	2.24	2.04
WGS 84 Gravity Anomaly and Geoid Height Matrices Only	41	1.47	1.22	0.96
	36	1.55	1.31	1.07
	30	1.67	1.45	1.21
	24	1.83	1.62	1.38
	18	2.09	1.90	1.71

n is the number of squares included in the sample out of a possible 4584 values (worldwide coverage).

* Least squares solutions made to the degree and order indicated using the WGS 84 data set (normal equation matrix).

Table 5.7

Comparison of 5°x5° Equal-Area Mean Geoid Heights
Computed From Earth Gravitational Models With Those
Derived From SEASAT-1 Geoid Height Data
(Units = Meters)

Earth Gravitational Model	Truncation Level (Degree)	RMS Differences		
		40% Obs n = 1108	80% Obs n = 993	100% Obs n = 887
WGS 84	41	± 2.34	± 1.24	± 0.89
	36	2.35	1.28	0.94
	30	2.41	1.36	1.04
	24	2.45	1.46	1.15
WGS 84 Experimental Earth Gravitational Models				
WGS 84*	36*	2.35	1.26	0.92
	30*	2.41	1.34	1.00
	24*	2.50	1.45	1.12
WGS 84 Less Geoid Height Matrix Data	41	2.65	1.67	1.38
	36	2.65	1.69	1.40
	30	2.68	1.74	1.46
	24	2.71	1.80	1.53
	18	2.84	1.98	1.72
WGS 84 Gravity Anomaly and Geoid Height Matrices Only	41	2.29	1.20	0.80
	36	2.31	1.23	0.85
	30	2.35	1.31	0.94
	24	2.40	1.41	1.07
	18	2.58	1.67	1.38

n is the number of squares included in the sample out of a possible 1654 values (worldwide coverage).

* Least squares solutions made to the degree and order indicated using the WGS 84 data set (normal equation matrix).

Table 5.8

Comparison of 10°x10° Equal-Area Mean Geoid Heights Computed From Earth Gravitational Models With Those Derived From SEASAT-1 Geoid Height Data (Units = Meters)

Earth Gravitational Model	Truncation Level (Degree)	RMS Differences		
		40% Obs n = 1421	80% Obs n = 1238	100% Obs n = 1036
WGS 84	41	± 3.31	± 2.04	± 0.84
	36	3.31	2.04	0.84
	30	3.32	2.05	0.86
	24	3.30	2.06	0.89
WGS 84 Experimental Earth Gravitational Models				
WGS 84*	36*	3.31	2.05	0.84
	30*	3.29	2.04	0.85
	24*	3.28	2.05	0.86
WGS 84 Less Geoid Height Matrix Data	41	3.41	2.19	1.16
	36	3.41	2.19	1.17
	30	3.42	2.20	1.18
	24	3.39	2.20	1.18
	18	3.45	2.29	1.22
WGS 84 Gravity Anomaly and Geoid Height Matrices Only	41	3.33	2.02	0.74
	36	3.33	2.01	0.74
	30	3.34	2.03	0.76
	24	3.33	2.04	0.80
	18	3.37	2.16	0.98

n is the number of squares included in the sample out of a possible 416 values (worldwide coverage).

* Least squares solution made to the degree and order indicated using the WGS 84 data set (normal equation matrix).

Table 5.9

Number of Satellites Used in EGM Development That are Sensitive to WGS 84 Gravitational Coefficients (Excluding Lumped Data Set)
(Orbital Perturbations $\geq$0.5 Meter)

Degree	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
2		10																																							
3	9	10	9																																						
4	9	10	9	10																																					
5	8	9	8	8	7																																				
6	8	8	8	9	8	8																																			
7	7	9	6	8	6	9	6																																		
8	8	8	8	8	8	8	7	4																																	
9	7	6	5	6	5	6	8	7	6																																
10	8	8	8	7	8	7		2	4	3																															
11	6	3	4	1		2	6	4	8	7	6																														
12	8	8	7	8	5	5	5	2		3	5	4																													
13	5	3				1		2	5	6	7	7	8																												
14	8	7	6	5	3	5	4	1			1	7	8	6																											
15	4	1						2	3	3	3	7	8	7	5																										
16	7	6	6	4		1		1				3	8	6	3	2																									
17	3									1	2	7	8	8	6	5	3																								
18	7	5	3	3	1							2	6	7	3	2																									
19												4	8	6	5	6	5	2																							
20	4	2	1	2								1	3	7	4																										
21												1	7	7	7	3	1	3	1	1																					
22	1	2										1	2	5	1																										
23												1	3	5	4	3	2	2																							
24		1										1	1	4	1																										
25												1	2	6	3	2	1																								
26	2											1	1	4																											
27												1	2	6													1														
28													1	3	1									1		1	2	2													
29													1	3	2									1			1	1													
30												1	1	2												1	3	3	2	1											
31												1	1	4													2		1												
32												1												1			3	3	3	1											
33													1	2																											
34													1														3	2	3	1											
35														3	1												1														
36																											2	1	3												
37															1																										
38																											2		3												
39																																									
40																											1		1												
41														1																											

Order

Table 5.10

Correlation Matrix
- WGS 84 Earth Gravitational Model Coefficients -

X = Correlation Coefficients ≥ 0.5
✱ = Correlation Coefficients ≥ 0.7

Degree	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
2																																										
3	✱	✱	X																																							
4	✱	✱	✱	X																																						
5	✱	✱	✱	X																																						
6	✱	✱	✱	✱	X																																					
7	✱	✱	✱	X	X	X																																				
8	✱	✱	✱	✱	✱		X																																			
9	✱	✱	✱	✱	X	X																																				
10	✱	✱	✱	✱	✱	X	X																																			
11	✱	✱	✱	✱	X	X	X																																			
12	✱	✱	✱	X	X	X	X	X																																		
13	✱	✱	✱	✱	X	X	X						✱	✱																												
14	✱	✱	✱	X	X	X	X	X						X																												
15	✱	X	✱	X	X		X						✱	X	✱	✱																										
16	✱	✱	X	X	X									X	X	X																										
17	✱	X	X											✱	✱	✱	✱																									
18	✱	X	X												X	X		X																								
19	✱	X												✱	X	✱	✱	X	X																							
20	X	X	X												✱	X		X																								
21	✱														X	✱			X																							
22	X		X											X	✱	X																										
23	X														X	X	X	X																								
24	X													X	✱	X																										
25	X														X	X																										
26	X													X	✱	X									X																	
27															X	X	X										✱															
28	X													X	✱										X		X	✱														
29															X	X											✱	X														
30															X												X	✱	✱													
31															X												X		✱													
32																												✱		✱												
33																												X	✱													
34																											X	X	✱	✱												
35	X																										X	X														
36																											X	X	X													
37																											X															
38																											X	X														
39																																										
40															X																											
41																																										

Order

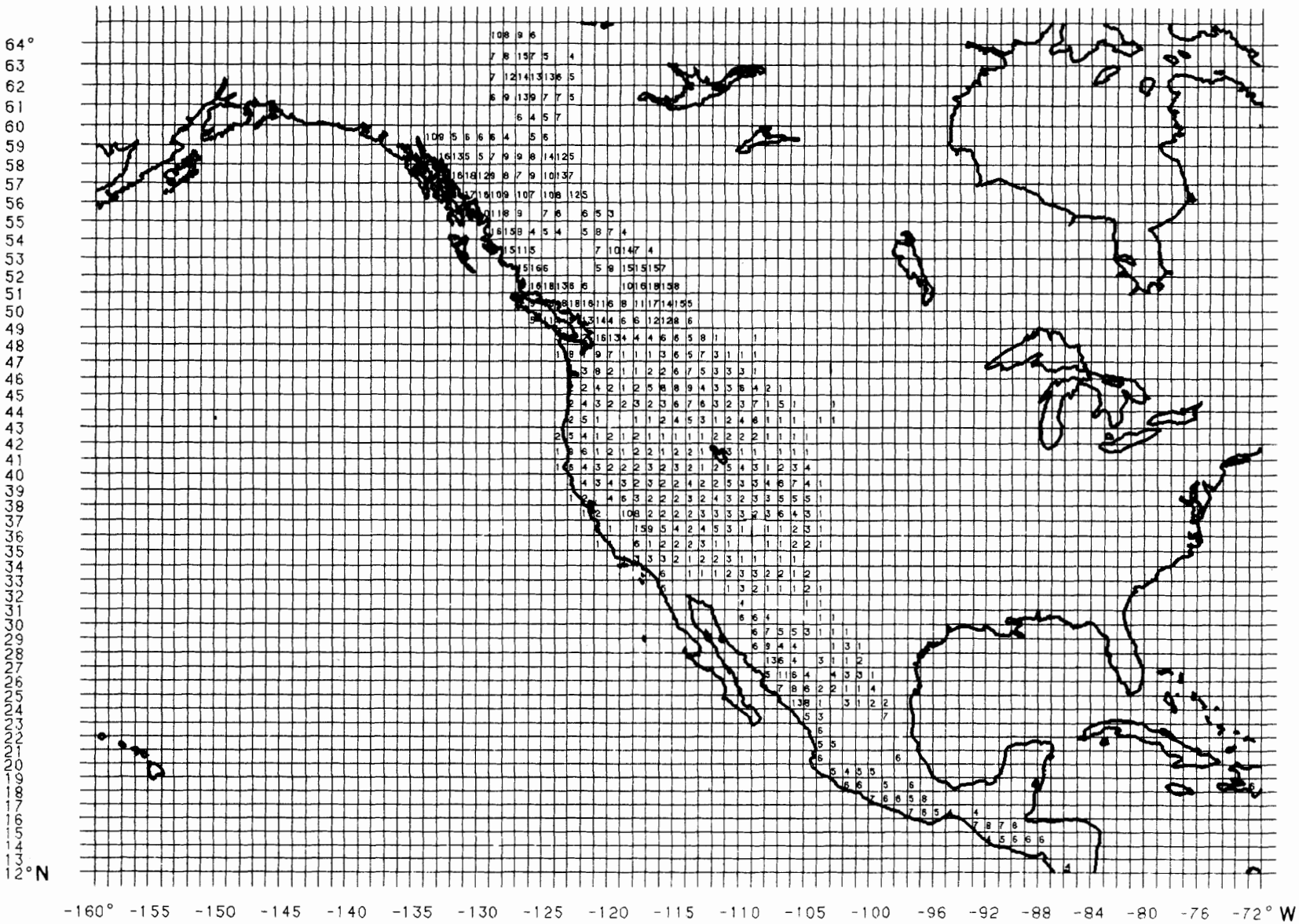

Figure 5.1. 1°x1° Mean Terrain Corrections Used in Development of WGS 84 EGM (Units = Milligals)

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